Essentials of Geophysics 12.201/501
Problem Set 6

Assigned Monday October 28
Due Wednesday November 6

1. Consider a spherical harmonic expansion of the magnetic potential

$$V = \frac{a}{\mu_0} \sum_{l=1}^{\infty} \left(\frac{a}{r}\right)^{l+1} \sum_{m=0}^{l} (g_l^m \cos m\lambda + h_l^m \sin m\lambda) P_l^m(\cos\theta) \qquad (1)$$

   (a) Which Gauss coefficients describe the dipole component of V ?

   (b) Show that with the normalization factor a/μ_0 of the coefficients have the dimension of the field.

   (c) Show that the components of the dipole moment of the Earth along the geographic axes are proportional to g_1^0, g_1^1 and h_1^1.

   (d) Find an expression for the angle between the magnetic and geographic axes in terms of the Gauss coefficients. Determine the angle with the information given in table 7.1 in Stacey.

   (e) Using the time derivatives of the Gauss coefficients from table 7.1 in Stacey, does the dipole field currently increase or decrease in strength? What is the reorganization time of the dipole field and how much is the rate of change (say in per cent per 100yr). What does this mean?

2. Suppose the Earth's magnetic field in the core can be represented as a sum of harmonics

$$\mathbf{B} = \sum_{n=1}^{\infty} \mathbf{B}_n e^{\alpha t} e^{2\pi i n x/L} \qquad (2)$$

   in which the coefficients $\mathbf{B}_n$ are constant and L$\approx$1000 km, the longest realistic wavelength in the core.

   (a) If there is no mechanism for regenerating the field (i.e., there is no dynamo action $\rightarrow$ no advection term in the equation that describes the variation of the field with time - derived in class), what would be the expression for α?

(b) What would be the longest possible decay time for any component of the field, using realistic values for physical properties of the core?

3. Laboratory analysis of the paleomagnetic direction in rock samples of Pennsylvanian age from South America indicates that their magnetic inclination is 70° and their declination is 83°. The samples were taken at -20°, -55°.

   (a) Find the location of the virtual geomagnetic pole that would produce this magnetization direction.

   (b) What can you say about the plate tectonic drift of the South American continent since Pennsylvanian time given this paleopole? (discuss magnitudes plus rates of any migrations and rotations.)

4. Show how the definition of the magnetic inclination $\tan I = \frac{Z}{H}$ can be used to obtain the differential equation for a magnetic line of force. Solve this equation and plot a few of those lines.

5. ESA scientists launch three probes with magnetometers measuring $|\mathbf{B}|$ to determine the magnetic field of the planet Krypton. Unfortunately all of the engines in the devices fail and each probe falls into the planet along a straight line. The three trajectories are found to be mutually orthogonal lines passing through the center of the planet.

   (a) Using the information given below, describe as completely as possible Krypton's magnetic field (hint, use spherical harmonics; first, determine the basic property of Krypton's field from the spatial attenuation of $|B_i(R)|$).

   (b) discuss any ambiguity involved in your answer.

   Information: all of the probes found that $|B_i(R)| = K_i/R^3$, $R > R_0$ where R is measured from the center of Krypton.

   $$K_1 = 6 \times 10^{14} \text{ mWb}$$
   $$K_2 = 5 \times 10^{14} \text{ mWb}$$
   $$K_3 = 4 \times 10^{14} \text{ mWb}$$

   (NB magnetic induction B is measured in Tesla (T) or Wbm^{-2})