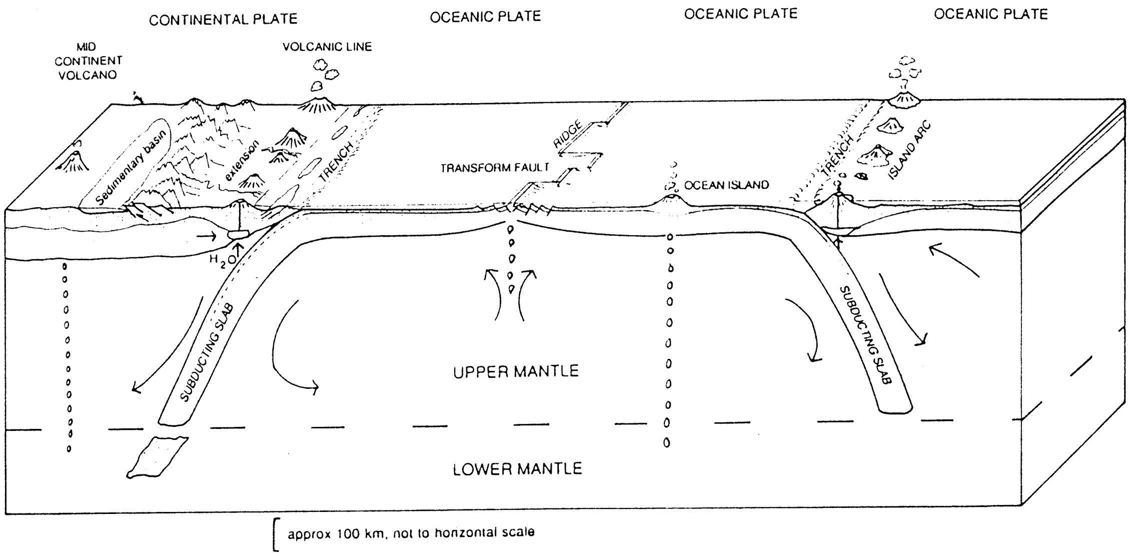
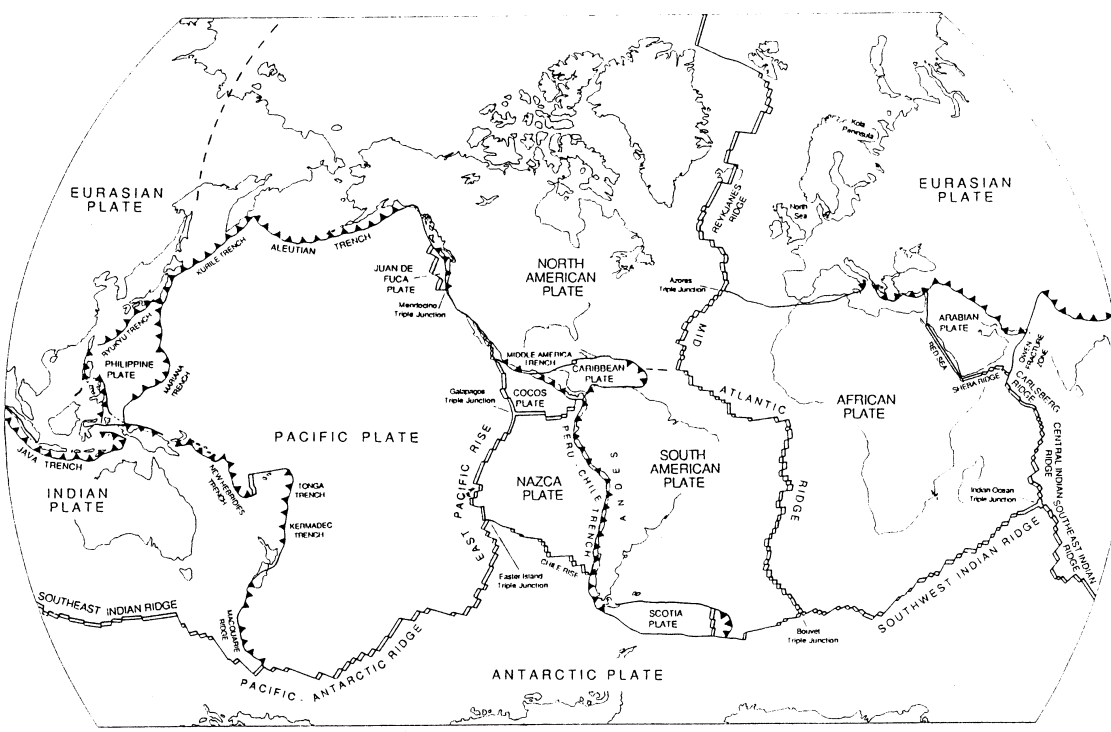

CONTINENTAL PLATE
OCEANIC PLATE
OCEANIC PLATE
OCEANIC PLATE
MID CONTINENT VOLCANO
VOLCANIC LINE
Sedimentary basin
extension
TRENCH
RIDGE
TRANSFORM FAULT
OCEAN ISLAND
TRENCH
ISLAND ARC
H2O
SUBDUCTING SLAB
UPPER MANTLE
SUBDUCTING SLAB
LOWER MANTLE
approx 100 km, not to horizontal scale


EURASIAN PLATE
EURASIAN PLATE
KURILE TRENCH
ALEUTIAN TRENCH
JUAN DE FUCA PLATE
Mendocino Triple Junction
NORTH AMERICAN PLATE
REYKJANES RIDGE
Kola Peninsula
North Sea
Azores Triple Junction
RYUKYU TRENCH
PHILIPPINE PLATE
MARIANA TRENCH
MIDDLE AMERICA TRENCH
CARIBBEAN PLATE
Galapagos Triple Junction
COCOS PLATE
MID ATLANTIC RIDGE
ARABIAN PLATE
RED SEA
SHEBA RIDGE
OWEN FRACTURE ZONE
CARLSBERG RIDGE
AFRICAN PLATE
PACIFIC PLATE
JAVA TRENCH
NEW HEBRIDES TRENCH
TONGA TRENCH
KERMADEC TRENCH
EAST PACIFIC RISE
NAZCA PLATE
PERU - CHILE TRENCH
ANDES
SOUTH AMERICAN PLATE
CENTRAL INDIAN RIDGE
Indian Ocean Triple Junction
SOUTHEAST INDIAN RIDGE
INDIAN PLATE
Easter Island Triple Junction
CHILE RISE
SOUTHEAST INDIAN RIDGE
MACQUARIE RIDGE
SCOTIA PLATE
Bouvet Triple Junction
SOUTHWEST INDIAN RIDGE
PACIFIC - ANTARCTIC RIDGE
ANTARCTIC PLATE

Thermal Diffusion

$\Delta H = \rho C_p \Delta T$ internal energy change per unit volume

$$\left.\begin{array}{l}\rho = \text{mass density} \simeq 3.4 \times 10^3 kg/m^3 \\ C_p = \text{specific heat} \simeq 0.71 J/g/K\end{array}\right\} \text{for mantle silicate rocks}$$

Heat flows down temperature gradients:

$$Q = k\tfrac{\partial T}{\partial z} = \text{heat flux} \quad \text{Fourier's Law, 1D}$$
$$k = \text{thermal conductivity} \simeq 2.5 W/m/K$$

Energy balance: A = heat production per unit volume

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial Q}{\partial z} + A \qquad \text{1D, no motion}$$
$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p v_z \frac{\partial T}{\partial z} = \frac{\partial Q}{\partial z} + A \qquad \text{1D, with motion}$$
$$\rho C_p (\frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla} T) = -\vec{\nabla} \cdot \vec{q} + A \qquad \text{3D, with motion}$$

Fourier's Law (3D): $\vec{q} = -k\vec{\nabla}T$

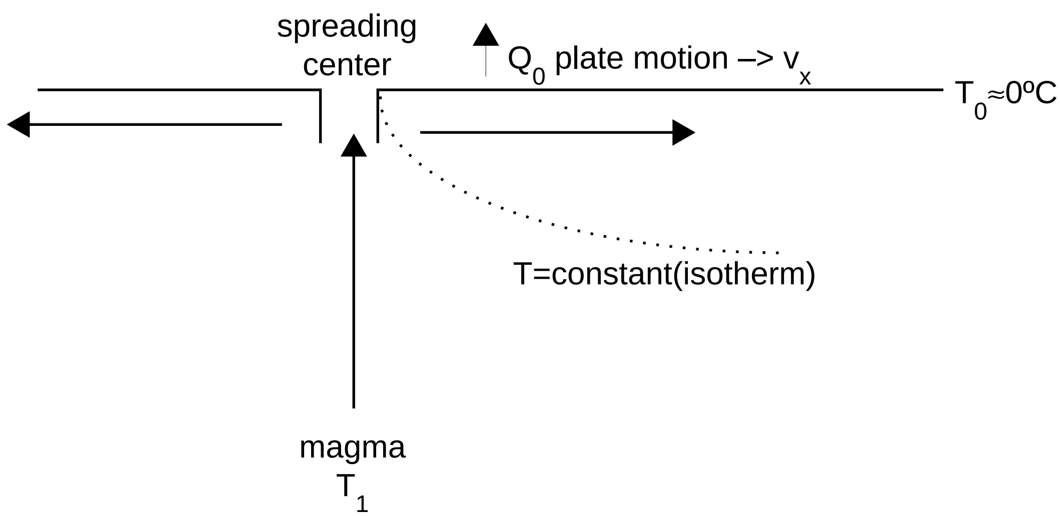
Thermal diffusion equation:

$$\frac{dT}{dt} = \kappa \nabla^2 T + A$$
$$\kappa = k/\rho C_p \simeq 10^{-6} m^2/s = \text{thermal diffusivity}$$
$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} = \text{material derivative}$$

$\vec{v} \cdot \vec{\nabla}$ is the advective term.

Cooling of Oceanic Lithosphere, I

Consider the 2D, steady-state problem:



This picture does not change with time!

Thermal diffusion equation: $\dfrac{dT}{dt} = \kappa \nabla^2 T + A$

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla} T \approx v_x \frac{\partial T}{\partial x} \quad \text{horizontal motion}$$

$$\kappa \nabla^2 T \simeq \kappa \frac{\partial^2 T}{\partial z^2} \quad \text{ignore horizontal conduction}$$

$$A \simeq 0 \quad \text{ignore heat production}$$

Define $\tau = x/v_x$ = seafloor age

$$\frac{\partial T}{\partial \tau} = \kappa \frac{\partial^2 T}{\partial z^2}$$

$$T = T(z, \tau)$$

Cooling of Oceanic Lithosphere, II

We need to solve

$$\frac{\partial T}{\partial \tau} = \kappa \frac{\partial^2 T}{\partial z^2}$$

subject to the boundary conditions:

$$T(z, 0) = T_1$$
$$T(0, \tau) = T_0 \simeq 0°C$$

The solution is an error function:

$$T(z, \tau) = T_1 \mathrm{erf}(z/2\sqrt{\kappa\tau})$$

$$\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$$

The surface heat flux is:

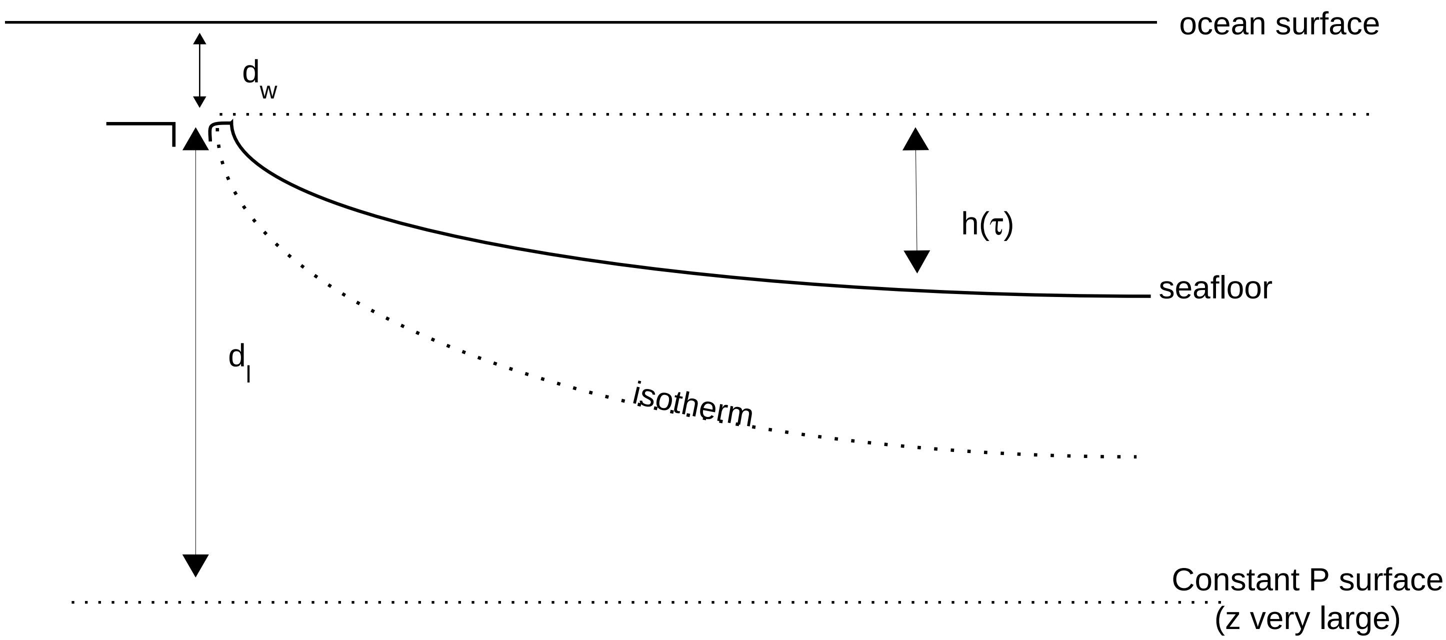
$$Q_0(\tau) = k \left(\frac{\partial T}{\partial z}\right)_0 = \frac{kT_1}{\sqrt{\pi\kappa\tau}} \sim \tau^{-1/2}$$

There will also be a topographic effect, because density depends on temperature:

$\rho(T) = \rho_0(1 - \alpha T)$

α = coefficient of thermal expansion $\simeq 3.3 \times 10^{-5}/K$

Cooling of Oceanic Lithosphere, III



Principle of Isostacy: at great depth, surfaces of constant pressure will be horizontal $\Longrightarrow$ mass above this "depth of compensation" will be approximately constant.

$$P(z) = \int_0^z \rho(\varphi) g(\varphi) d\varphi \simeq g_0 \int_0^z \rho(\varphi) d\varphi = \text{constant}$$

Mass balance:

$$M = \rho_w d_w + \rho_1 d_l \qquad \text{at ridge crest}$$

$$\begin{aligned} M &= \rho_w (d_w + h) + \rho_1 \int_{d_w+h}^{d_w+d_l} (1 + \alpha(T_1 - T)) dz \\ &= \rho_w (d_w + h) + \rho_1 (d_l - h) + \alpha \rho_1 \int_{d_w+h}^{d_w+dl} (T_1 - T) dz \end{aligned}$$

Equating:

$$(\rho_1 - \rho_w) h = \alpha \rho_1 \int_{d_w+h}^{d_w+d_l} (T_1 - T) dz \simeq \alpha \rho_1 \int_0^\infty (T_1 - T) dz$$

Cooling of Oceanic Lithosphere, IV

Isostacy thus implies:

$$h(\tau) = \left(\frac{\rho_1}{\rho_1 - \rho_w}\right) \alpha T_1 \int_0^\infty [1 - \mathrm{erf}(z/2\sqrt{\kappa\tau})]dz$$

But we know

$$\int_0^\infty [1 - \mathrm{erf}(\frac{x}{a})]dx = a/\sqrt{\pi}$$

Therefore,

$$h(\tau) = \left(\frac{\rho_1}{\rho_1 - \rho_w}\right) \alpha T_1 2\sqrt{\frac{\kappa\tau}{\pi}}$$

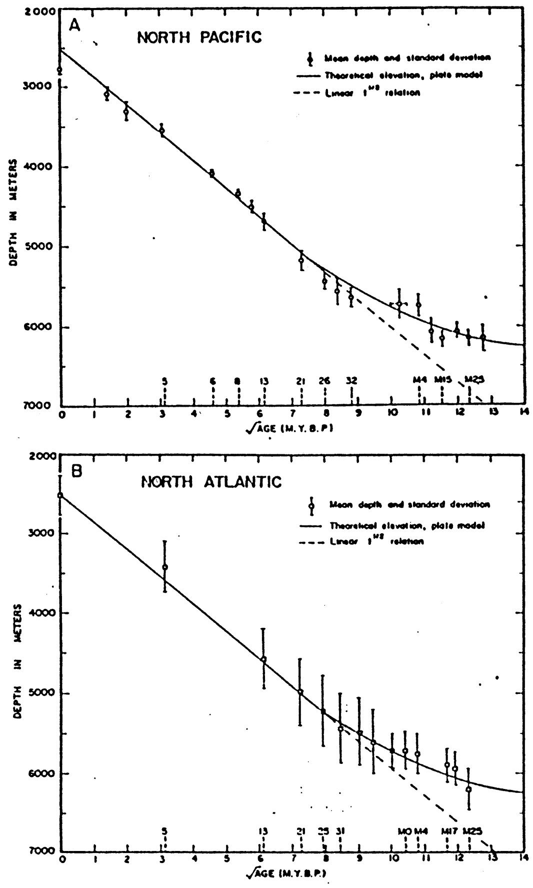
Assuming $T_1 = 1300°C$, we obtain:

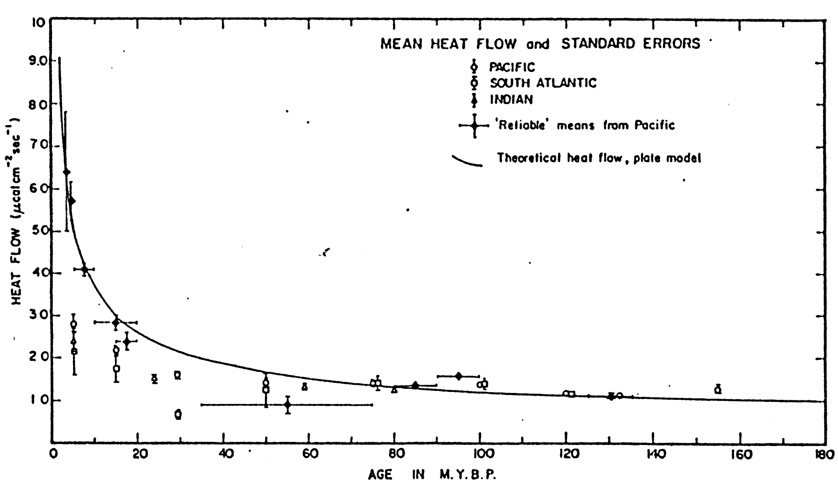
$$h(\tau) \simeq 385\sqrt{\tau}$$

$$[h]\text{=meters}, \qquad [\tau]\text{=My}$$

At the ridge crest, $d_w \simeq 2500$ m

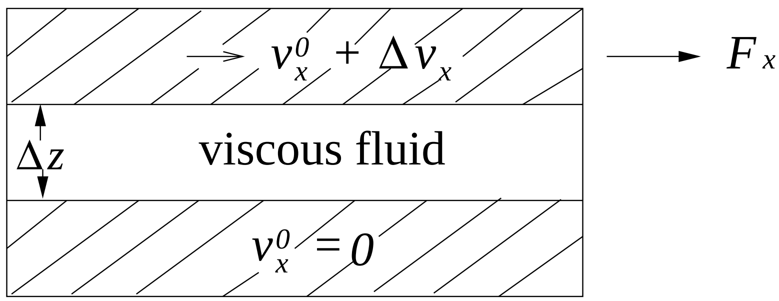
Therefore, $\quad d(\tau) \simeq 2500 \text{ m} + 340\sqrt{\tau \text{ My}}$


A
NORTH PACIFIC
Mean depth and standard deviation
Theoretical elevation, plate model
Linear $t^{1/2}$ relation
DEPTH IN METERS
√AGE (M.Y.B.P.)
B
NORTH ATLANTIC
Mean depth and standard deviation
Theoretical elevation, plate model
Linear $t^{1/2}$ relation
DEPTH IN METERS
√AGE (M.Y.B.P.)


MEAN HEAT FLOW and STANDARD ERRORS
PACIFIC
SOUTH ATLANTIC
INDIAN
'Reliable' means from Pacific
Theoretical heat flow, plate model
HEAT FLOW (μcal cm^{-2} sec^{-1})
AGE IN M.Y.B.P.

Newtonian Viscosity

Consider a thin layer of viscous material sandwiched between two rigid, parallel plates of area A:



AreaA_z

In steady state we observe

$$\frac{F_x}{A_z} \sim \frac{\Delta v_x}{\Delta z}$$

Newtonian (linear) viscosity law:

$$\begin{aligned}
\tau_{xz} &= \eta \frac{\partial v_x}{\partial z} \\
\tau_{xz} &= xz \text{ stress } [N/m^2 = Pa] \\
\frac{\partial v_x}{\partial z} &= xz \text{ strain rate } [s^{-1}] \\
\eta &= \text{dynamic viscosity [Pas]}
\end{aligned}$$

Generalization to 3D: $x = x_1, y = x_2, z = x_3$

$$\tau_{ij} = 2\eta\dot{\epsilon}_{ij} - p\delta_{ij}$$

$$\dot{\epsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \qquad i,j = 1,2,3$$

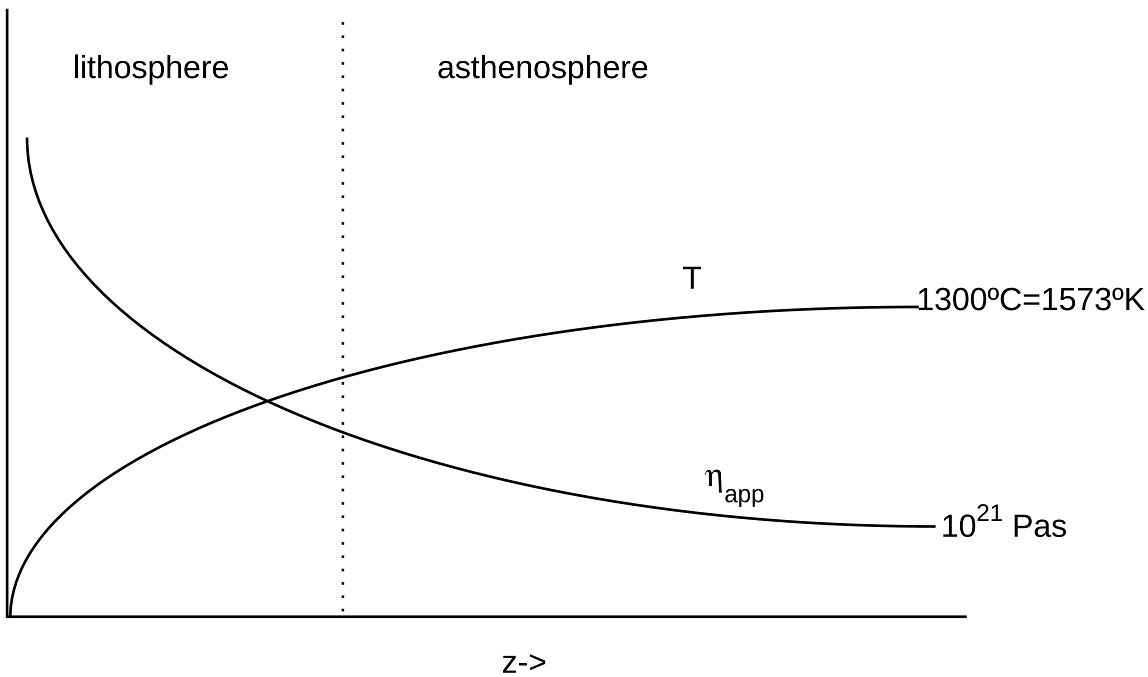
Mantle Viscosity

For mantle rocks (peridotites), strain rate and stress are nonlinearly related:

$$\dot{\epsilon} \sim \tau^3$$

Moreover, the relationship is strongly temperature dependent. For dry olivine,

$$\begin{aligned} \dot{\epsilon} &= C\tau^3 e^{-E^*/RT} \\ C &\simeq 4.2 \times 10^5 MPa^{-3}s^{-1} \\ E^* &\simeq 5.2 \times 10^5 J/mol \text{ activation energy} \\ R &\simeq 8.317 J/^\circ K/mol \text{ gas constant} \end{aligned}$$



$$\eta_{app} = \frac{e^{E^*/RT}}{C\tau_{ref}^2}$$

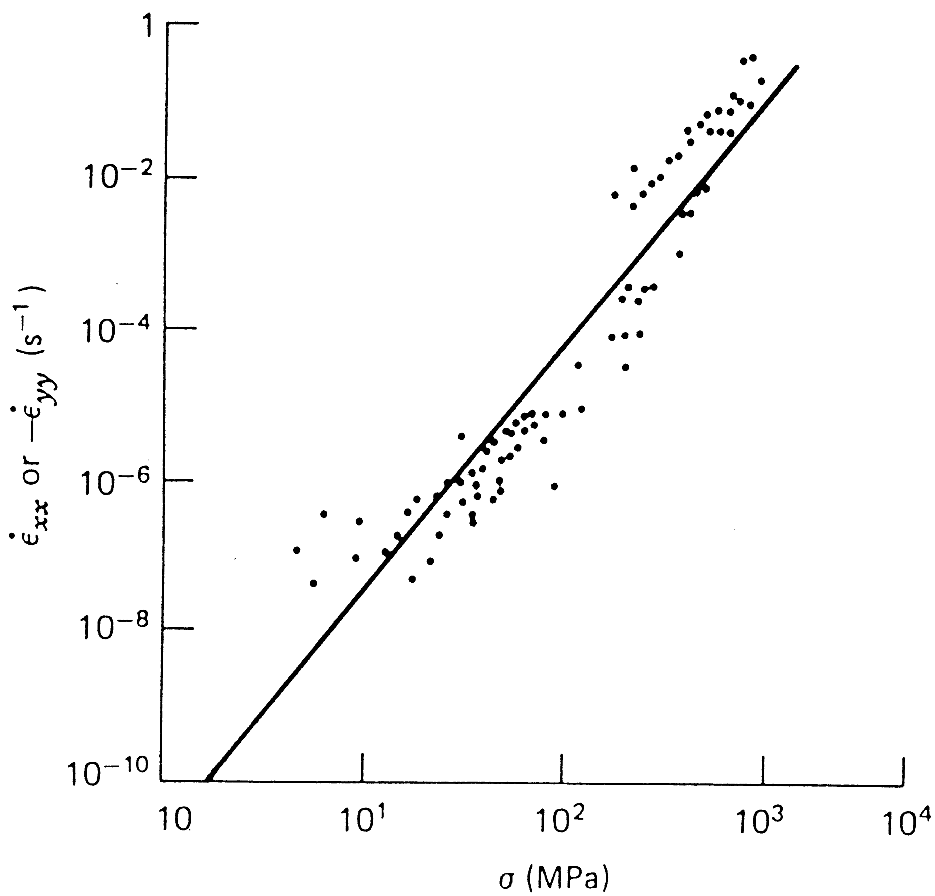
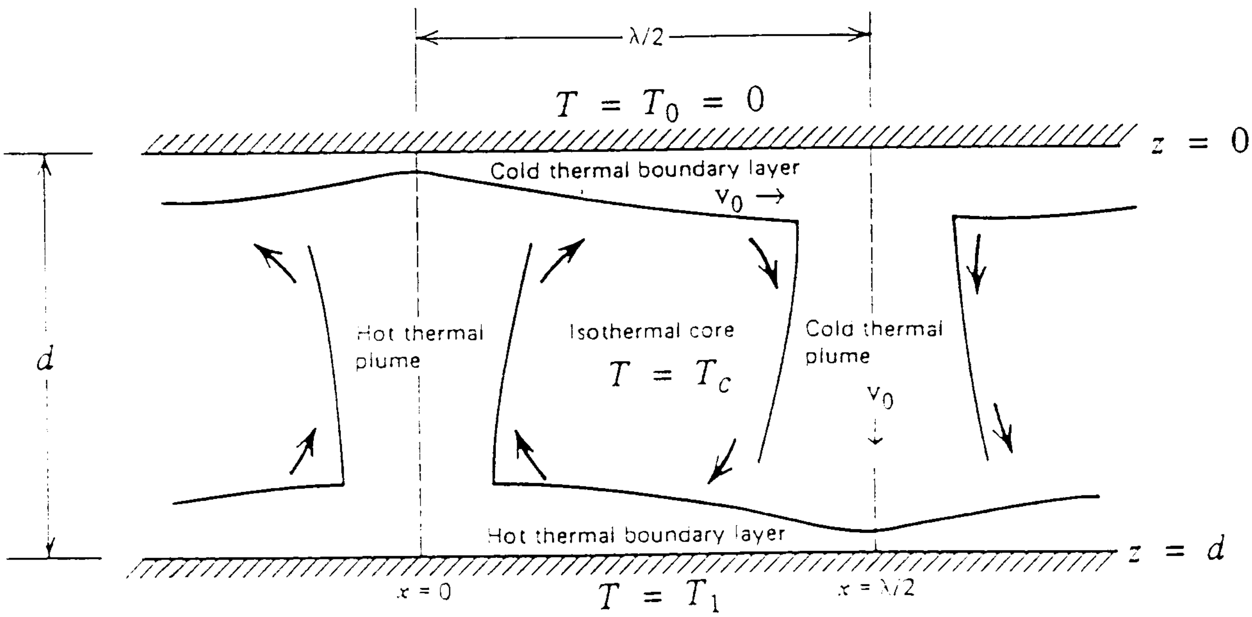


Figure 7-19 Observed dependence of strain rate on stress for olivine at a temperature of 1400°C. (After M. F. Ashby and R. A. Verall, Micromechanisms of flow and fracture and their relevance to the rheology of the upper mantle, *Philosophical Transactions of the Royal Society, London*, **288A**, 59–95, 1977.)

Analytical Toy Model of Mantle Convection

Approximations: "Boundary-Layer Theory" (2D)

- Periodic structure in x with wavelength λ
- Only driving forces are buoyancy forces of sinking slab and rising plume, which are symmetric
- Only resistive forces are viscous drags in isothermal core, which are symmetric



	mass excess at "trench" per unit length	×	gravitational acceleration	×	length of slab
$F_{slab} \simeq$	$\rho_c \alpha T_c 2\sqrt{\frac{\kappa\lambda}{2\pi v_0}}$		g		d

$$T_c = T_0 + (T_1 - T_0)/2 = T_1/2$$

$$F_{slab} = \rho_c g d \alpha T_1 \sqrt{\frac{\kappa\lambda}{2\pi v_0}}$$

To calculate the resistive forces due to viscous drag, we approximate the velocity gradients as constants:

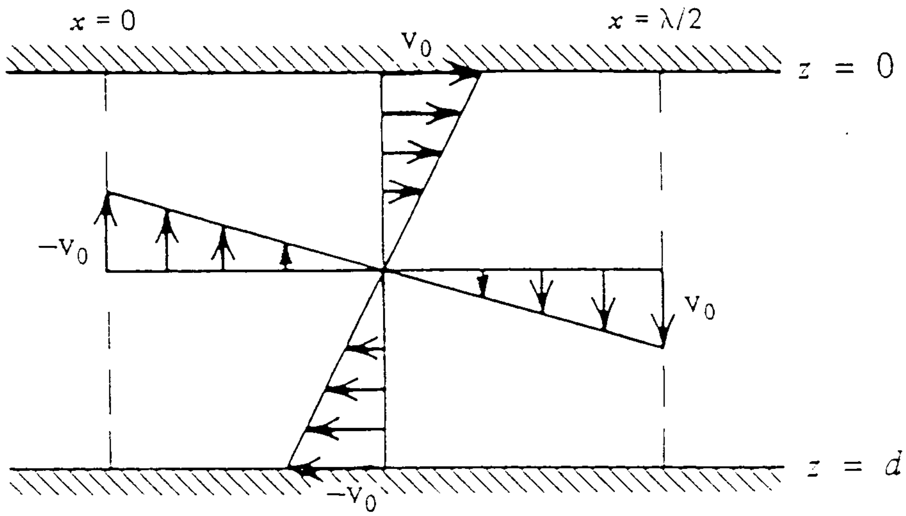


Plate:

$$v_x = v_0(1 - \frac{2z}{d}) \ , \ \tau_{plate} = \eta \frac{\partial v_x}{\partial(-z)} \simeq \eta \frac{2v_0}{d}$$

Slab:

$$v_z = -v_0(1 - \frac{4x}{\lambda}) \ , \ \tau_{slab} = \eta \frac{\partial v_z}{\partial x} \simeq \eta \frac{4v_0}{\lambda}$$

Force balance requires:

$$2F_{slab} = 2d\tau_{slab} + 2(\frac{\lambda}{2})\tau_{plate}$$

Mass balance requires:

$$u_0 \frac{\lambda}{2} = v_0 d \Longrightarrow \frac{\lambda}{2} = d \text{ if } u_0 = v_0$$