So far, have $(C-A)$ in terms of J_2 or C_2^0
We have 5 $l=2$ terms, 6 terms in $\underset{\approx}{I}$.
What is value of C?

Recall

$$C = I_{zz} = \iiint (x^2+y^2)\, dM$$

Angular momentum

$$\underset{\sim}{L} = \underset{\approx}{I}\,\underset{\sim}{\omega}$$

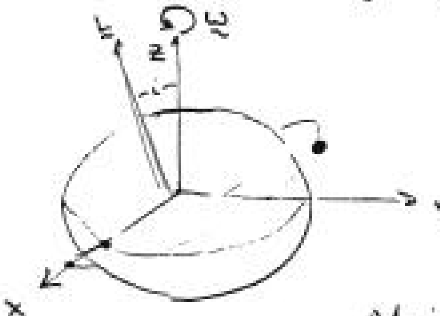
$\underset{\approx}{I}$ inertia tensor

$\underset{\sim}{\omega}$ rotation vector

to 1st order, $\underset{\sim}{\omega} \parallel z$

$$L = C\omega$$

Recall satellite geodesy – earth exerts torque $\underset{\sim}{T}$ on satellites, $\dot{\underset{\sim}{L}} = \underset{\sim}{T}$



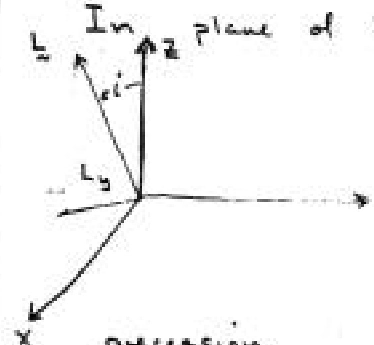
$$\bar{T}_x = \frac{3}{2}\frac{GMma^2}{r^3}J_2 \sin i \cos i$$

Newton's 3rd ⇒

T acts on earth too!

Major components – lunar, solar

In plane of ecliptic



$$L_y = \omega C \sin i$$

$$\delta L_x = \bar{T}_x\, \delta t = \frac{2\pi}{\omega_D}\cdot\frac{3}{2}\frac{GM m_D a^2}{r_D^3}J_2 \sin i \cos i$$

Kepler ⇒ $\frac{GM}{r_D^3} = \omega_D^2$, $J_2 = \frac{C-A}{Ma^2}$

<u>precession</u>

$$\omega_{PL} = \frac{1}{2\pi}\frac{\delta L_x}{L_y} = \frac{3}{2}\frac{C-A}{C}\frac{m_D}{M}\omega_D \cos i$$

Also — solar torque, planet torques $T \sim 26{,}000$ yrs

dynamic ellipticity

$$H = \frac{C-A}{C} = 0.00327364 = \frac{1}{305.47}$$

Note — not the same as f

$H < f$

internal surfaces not as elliptical as surface

core precession "different"

$$C = \frac{J_2}{H} \cdot Ma^2 = 0.3307\, Ma^2$$

(Shell - $0.6\ Ma^2$
Uniform $0.4\ Ma^2$)

Other motions

Due to $l=2$ components of gravity:

1) Nutation (T_{y}, T_{x} (vs $\overline{T}_x$)) (core response different ⇒ coupling)
2) Chandler wobble (free nutation) (wobbly football)

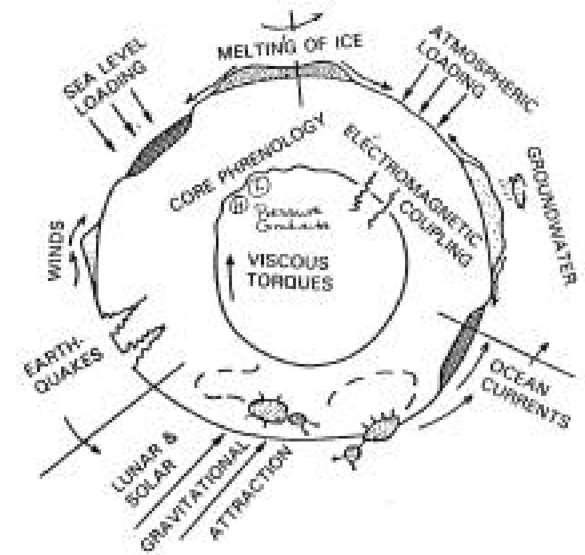
$$\dot{\underset{\sim}{L}} = (\underset{\approx}{I}\,\underset{\sim}{\omega})^{\cdot} = \dot{\underset{\approx}{I}}\,\underset{\sim}{\omega} + \underset{\approx}{I}\,\dot{\underset{\sim}{\omega}} = 0$$

see Garland, section 1.3; Stacey Section 3.3

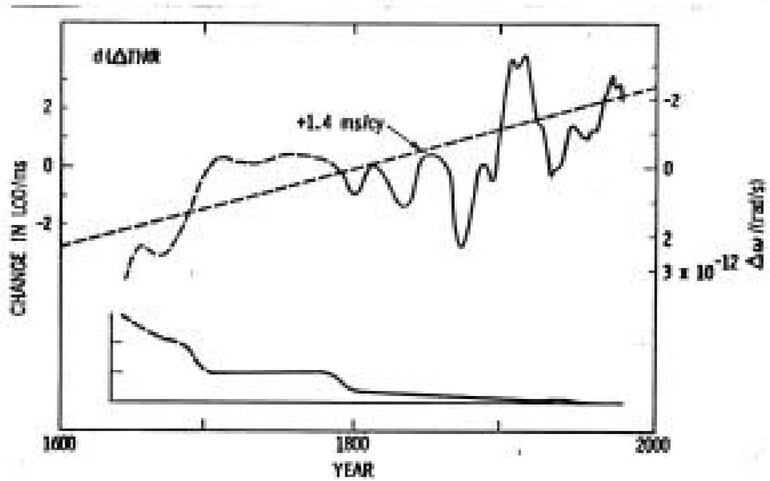
3) Polar wander —

Precession & Nutation — pole moves relative to stars ($\underset{\sim}{L}$ changes)

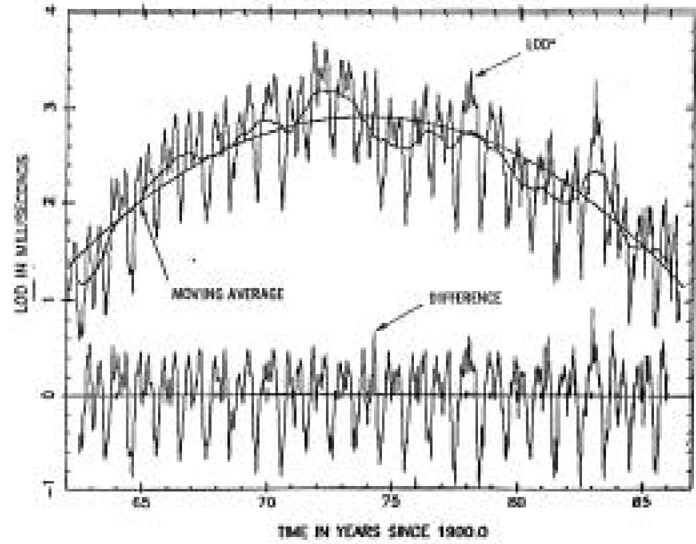
Chandler wobble & Polar wander — rotation axis moves relative to figure of earth.

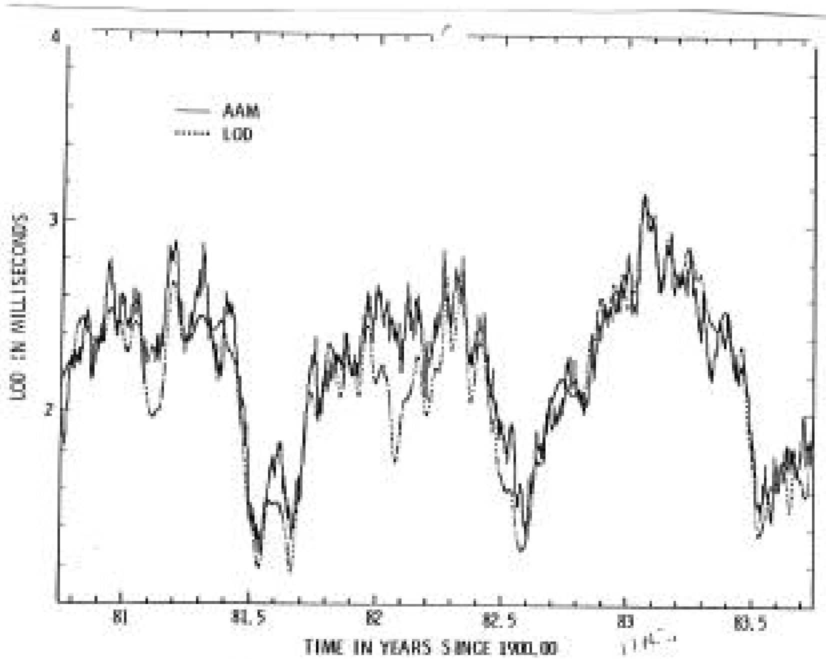


LENGTH OF DAY (LOD): 1620 TO 1980



COMBINED LOD* WITH 365 DAY MOVING AVERAGE BEST FIT QUADRATIC





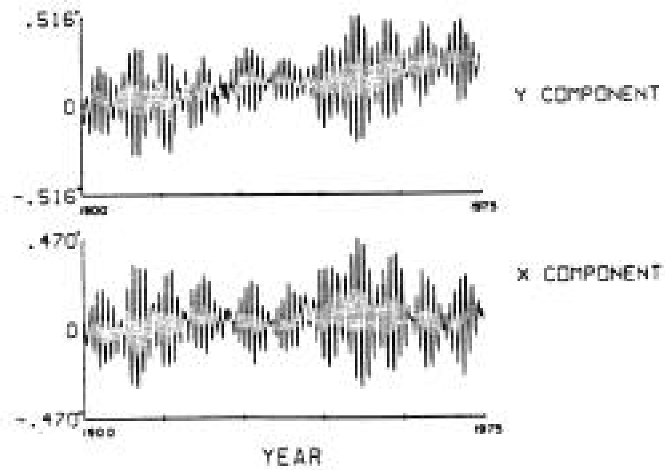


Figure 1. *x* and *y* components of the 1900–74 monthly variation-of-latitude data determined by the ILS-IPMS. They define the position of the instantaneous rotation pole in the CIO coordinate system.

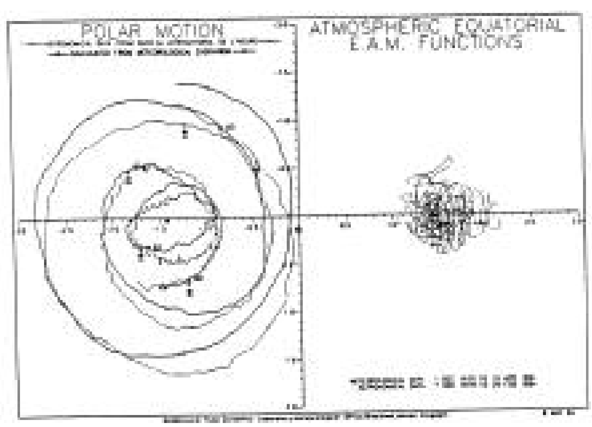


Figure 7

a) Observed polar motion as determined by the Bureau International de l'Heure compared with polar motion as inferred from the European Centre Meteorological data.
b) polar plot of the smoothed value. The units of both axis radians with 10^{-6} rad equivalent to a displacement of about 6.4 m over the Earth's surface. (Hide, 1985 and private communication.)