Essentials of Geophysics 12.201/501 2002 Homework 4, due October 11

Suggested reading: Chapter 5 in Turcotte and Schubert.

1) Start by writing down a general formula for the variation of the gravitational acceleration with radius for a spherical body like the Earth, in which the density is a function of radius only.

- Now, derive and plot the expression for *g(r)* assuming $d\rho/dr=0$.

- Next, consider a two-step variation of density for a planetary body much like the Earth in the sense that it has a core. Derive a general expression for *g(r)* where $\rho=\rho_c$ for $0 \le r \le b$ and $\rho=\rho_m$ for $b \le r \le a$. Plug in densities and radii adequate for the Earth and plot the variation of g with r. You must obtain a curve similar to the one pictured in Fowler, Fig. 4.31, p. 111.

- In a planet, suppose that there is no variation of gravitational acceleration with increasing depth. What does this tell you about the distribution of density within the planet? Express your answer in terms of mean density and total radius.

2)

- Consider a sphere of radius R with a uniform density anomaly $\Delta\rho$, buried at depth z. Derive an expression for the surface gravity anomaly Δg, measured positively downward. Plot the gravity profile for varying x on the surface.

- Two identical spheres are buried at the same depth. Their densities are twice that of the surrounding material. Plot the surface anomalies to show what happens as the spheres are brought closer together. Plot the anomalies that result when both spheres are moved vertically.

- Discuss your results in terms of the *non-uniqueness* of the data. What can be determined from surface measurements of gravity anomalies?

3)

- Derive an expression for the gravity anomaly (measured positively downward) of an infinitely long horizontal cylinder of radius R with anomalous density $\Delta\rho$ buried at depth z beneath the surface, in terms of x, the horizontal distance from the surface measurement to the point on the surface directly over the cylinder axis.

- You are attempting to find the location and depth of a tunnel beneath a hillside. The tunnel has an approximately cylindrical cross-section. The enclosed map shows station elevation in meters and raw gravity in gravity units (10. g.u. = 1 milligal, 1 milligal = 10^{-5} ms $^{-2}$. Do all the necessary reductions to the data in order to find the location of the tunnel. You may ignore terrain corrections.

• Plot up a cross-section of the gravity anomaly from the tunnel along a line perpendicular to its strike. Using the formula for the gravity anomaly from a buried cylinder, what can you conclude about the depth and radius of the tunnel? Consider both air and water in the tunnel.

• Helpful hints. The Matlab functions, `mesh, meshgrid, plot, plot3, fmins`, and the concept of the `function`-environment (similar to `subroutine` in FORTRAN) will come in handy. You will want to minimize the root mean sqauare error between your measurements and the predicted gravity anomaly profile based on the cylinder model