Essentials of Geophysics 12.201/501
Problem Set 7

1. In an ideal fluid, there are no shear tractions on any plane. Show that the stress tensor $\mathbf{T}$ is given by

$$\mathbf{T} = -p\mathbf{I} \tag{1}$$

where p is the pressure.

2. (a) Show how $\sigma_{ij} = c_{ijkl}\epsilon_{kl}$ together with $c_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$ lead to

$$\sigma_{ij} = c_{ijkl}\epsilon_{kl} = \lambda\delta_{ij}\epsilon_{kk} + 2\mu\epsilon_{ij} = \lambda\delta_{ij}\Delta + 2\mu\epsilon_{ij} \tag{2}$$

(b) Verify how

$$\rho\ddot{\mathbf{u}} = (\lambda + 2\mu)\nabla(\nabla \cdot \mathbf{u}) - \mu(\nabla \times \nabla \times \mathbf{u}) \tag{3}$$

leads to

$$\rho\frac{\partial^2(\nabla \cdot \mathbf{u})}{\partial t^2} = (\lambda + 2\mu)\nabla^2(\nabla \cdot \mathbf{u}) \tag{4}$$

and

$$\frac{\partial^2(\nabla \times \mathbf{u})}{\partial^2 t} = \frac{\mu}{\rho}\nabla^2(\nabla \times \mathbf{u}) \tag{5}$$

3. Read Fowler's derivation of the expressions for the elastic parameters (Appendix 2) and use the definitions to answer the following questions.

(a) One of the simplest ways to determine the elastic constants of a rock is to measure its density and the travel times of P and S-waves across a small sample. Suppose that you cut a core 2 cm in diameter and 6 cm long out of a homogeneous hand specimen of mitmite. The weight of the sample is 61.45 g. A compressional impulse given at one end arrives at the other end after 8.6 μs; for

a shear impulse, you find a travel time of 14.5 μs. What (in S.I. units) are the Young's modulus E, the Poisson's ratio s, and the rigidity μ of mitmite?

(b) Consider two half spaces separated by a surface Σ. The material constants for the two media are as follows:

	P-wave speed [km/s]	Poisson's ratio [?]	ρ [g/cm^3]
med 1	5.6	0.20	2.7
med 2	8.1	0.30	3.2

A P-wave is incident from medium 2 at an angle of incidence of 25°. What type of waves are produced upon transmission and/or reflection? Why? Compute all the angles of incidence and draw all the reflected and refracted (transmitted) rays.

4. A three-component seismogram reveals the arrival of several phases between about 5 and 8 s (see postscript file hw7fig1.ps). Which of these arrivals are the P, S, and Sp arrivals? Can you see any other phases? If so, explain what it/they could be. Is the mantle part of S_p an SH or an SV wave?

   (NB The three components of the seismograms refer to the component of ground motion that is recorded: 'Z' records the vertical component of ground motion, whereas 'N' and 'E' record the North-South and East-West components of ground motion, respectively.

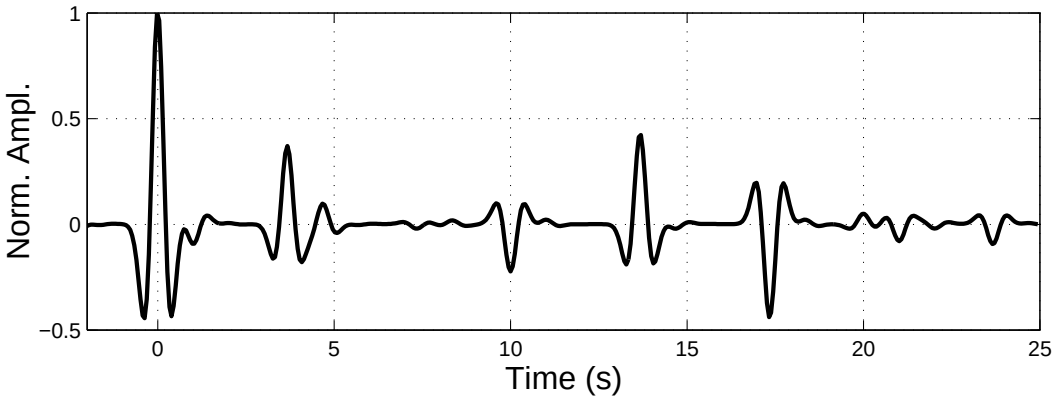
5. (a) Can Rayleigh waves exist in a half space? And Love waves? Why (not)? If they do, characterize their dispersion.

   (b) What type of surface wave is more likely to be caused by a nuclear explosion, a Rayleigh or a Love wave? Explain your answer.

   (c) Consider the sensitivity kernels of the surface waves as discussed in class. Are deep earthquakes, say deeper than 400 km, efficient in generating fundamental mode Rayleigh waves that can be used to study crustal structure? Can shallow earthquakes generate fundamental modes, higher modes, or both? Explain!

6. The following seismogram (calculated with a method known as the **reflectivity method**) shows the displacement due to a P-wave coming

in from a deep and far earthquake. The component shown is horizontal component of motion in the vertical plane containing the source and the station.



(a) Can you identify the phases?

(b) Assuming the Earth under the station is characterized by a homogeneous mantle and a crust on top of it, can you infer the crustal thickness, if you also know that the P-wave velocity in the crust 6 is km/s, and the crustal Poisson ratio is 1/4?

(c) Why did we use the horizontal seismogram rather than the vertical?