Scan of Fred Pollitz's Presentation

SCEC Crustal Deformation Modeling Workshop

Caltech, June 10-12, 2002

VISCO1D

- Semi-analytic program package for computing postseismic response to specified impulsive seismic sources
- Deformation field is a sum of viscoelastic normal modes
- Deformation computed as a sum of spherical harmonic components
- Accounts for effects of depth-dependent linear rheology and gravitational acceleration (g–effects, but not G–effects)
- Available by anonymous ftp at USGS

Semi-Analytic Postseismic Relaxation

1. Plane-layered elastic/viscoelastic model

2. Spherially-layered viscoelastic model (VISCO1D)

3. Comparisons among different approaches
   (i) VISCO1D with Rundle's code
   (ii) VISCO1D with TECTON
   (iii) VISCO1D with analytic result for infinitely long strike-slip fault (Nur and Mavko, 1974)

4. Aspherical viscoelastic model (VISCO3D)

Advantages of semi-analytic methods

- Fast and accurate computation for layered structures
- Spatial derivatives of deformation field are readily computed
- Inverse Laplace transform can be evaluated analytically (viscoelastic normal modes)
- Same computational effort yields postseismic deformation at arbitrary distance (and arbitrary time for viscoelastic normal modes)

Excitation by seismic source

$$\Phi_j^0(\hat{\mathbf{r}};s) = \left[\mathbf{M}(\mathbf{r}_0;s):\mathbf{E}_j(\mathbf{r}_0,\hat{\mathbf{r}};s) + \mathbf{F}(\mathbf{r}_0;s)\cdot\mathbf{E}_j'(\mathbf{r}_0,\hat{\mathbf{r}};s)\right] \varepsilon_j^{-1}\, \psi(s,s_j)$$

M: Laplace-transformed moment tensor at $\mathbf{r}_0 \sim 1/s$

F: Laplace-transformed force vector at $\mathbf{r}_0 \sim 1/s$

$\mathbf{E}_j$: Excitation tensor of Greens function response to moment tensor sources

$\mathbf{E}_j'$ Excitation tensor of Greens function response to single forces

$$\psi(s,s_j) = \begin{cases} 1 & j = \text{static mode} \\ (s+s_j)^{-1} & j = \text{viscoelastic mode} \end{cases}$$

Time Domain:

Static deformation $\sim H(t)$

Postseismic deformation $\sim [1 - \exp(-s_j t)]$

Spherically-layered viscoelastic medium

Viscoelastic problem

Displacement

$$\mathbf{u}^0(r,\hat{\mathbf{r}}\,t) = \frac{1}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty} ds\, e^{st} \sum_{j\,\in\,[l,m,n]} \mathbf{O}^j_{disp}\, \Phi^0_j(\hat{\mathbf{r}};s)$$

Spheroidal motion operator is

$$\mathbf{O}^S_{disp}(\mathbf{r}) = \left[U(r)\hat{\mathbf{r}} + V(r)\nabla_1\right]$$

Toroidal motion operator is

$$\mathbf{O}^T_{disp}(\mathbf{r}) = -W(r)\,\hat{\mathbf{r}} \times \nabla_1$$

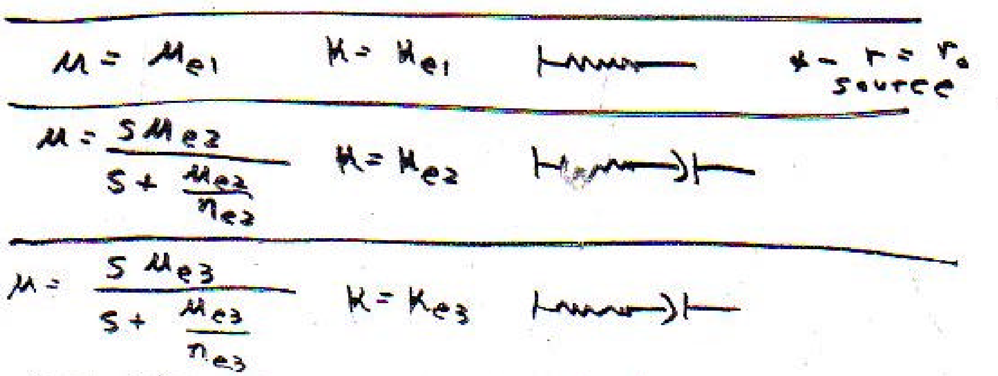
U, V: vertical and horizontal spheroidal mode displacement eigenfunctions

W: horizontal toroidal mode displacement eigenfunction

Traction

$$\hat{\mathbf{r}}\cdot\sigma(r,\hat{\mathbf{r}};t) = \frac{1}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty} ds\, e^{st} \sum_{j\,\in\,[l,m,n]} \mathbf{O}^j_{trac}\, \Phi^0_j(\hat{\mathbf{r}};s)$$

$$\mathbf{O}_{trac}{}^S(\mathbf{r}) = \left[R(r)\hat{\mathbf{r}} + S(r)\nabla_1\right]$$
$$\mathbf{O}_{trac}{}^T(\mathbf{r}) = -T(r)\,\hat{\mathbf{r}} \times \nabla_1$$



For a static mode j, ε_j is the kinetic energy integral:

$$\varepsilon_S = \omega_S^2 \int_0^a \rho(r) \left[U^2(r) + l_S(l_S+1)\, V^2(r)\right] r^2\, dr \qquad \text{(spheroidal modes)}$$

$$\varepsilon_T = \omega_T^2\, l_T(l_T+1) \int_0^a \rho(r)\, W^2(r)\, r^2\, dr \qquad \text{(toroidal modes)}$$

$\rho(r)$: radial density distribution

ω_j: angular frequency of the jth degenerate free oscillation on the spherical Earth model with total degree l_j

For a viscoelastic mode j, ε_j is:

$$\varepsilon_S = \int_0^a \left\{ \frac{1}{3}[2\partial_r U(r) - F]^2 + L\, r^{-2}\, [r\, \partial_r V(r) - V(r) + U(r)]^2 + r^{-2}\, (V(r))^2\, [2(L-1)\, L - L^2] \right\} \frac{\partial \mu_0(r,s)}{ds}\Big|_{s=-s_S} r^2\, dr$$

$$F = r^{-1}\, [2U(r) - L\, V(r)]$$

$$L = l_S(l_S+1)$$

$$\varepsilon_T = l_T(l_T+1) \int_0^a \left\{ [r\, \partial_r W(r) - W(r)]^2 + (l_T-1)(l_T+2)(W(r))^2 \right\} \frac{\partial \mu_0(r,s)}{ds}\Big|_{s=-s_T} dr \qquad (8)$$

a = Earth's radius

l_S, l_T = spherical harmonic degree of spheroidal or toroidal mode

For spheroidal motion the displacement-stress vector

$$\mathbf{y}_S(r,l) = \begin{pmatrix} U \\ R \\ V \\ S \end{pmatrix}$$

solves the system

$$\frac{d\mathbf{y}_S(r,l)}{dr} = \mathbf{A}_S(r,l)\,\mathbf{y}_S(r,l)$$

subject to 4 boundary conditions

for example $R(r_{\text{bottom}},l) = S(r_{\text{bottom}},l) = 0$

$R(\mathrm{a},l) = S(\mathrm{a},l) = 0$

r_{bottom}: arbitrary lower model boundary

For toroidal motion the displacement-stress vector

$$\mathbf{y}_T(r,l) = \begin{pmatrix} W \\ T \end{pmatrix}$$

solves the system

$$\frac{d\mathbf{y}_T(r,l)}{dr} = \mathbf{A}_T(r,l)\,\mathbf{y}_T(r,l)$$

subject to 2 boundary conditions

for example $T(r_{\text{bottom}},l) = 0$

$T(\mathrm{a},l) = 0$

Layer matrices

Spheroidal motion

$$\mathbf{A}_S(r,l) = \begin{pmatrix} -2\lambda\sigma^{-1}r^{-1} & \sigma^{-1} & \lambda\sigma^{-1}l(l+1)r^{-1} & 0 \\ -4\rho g r^{-1}+4\gamma r^{-2} & 2(\lambda\sigma^{-1}-1)r^{-1} & (-2\gamma r^{-2}+\rho g r^{-1})l(l+1) & l(l+1)r^{-1} \\ -r^{-1} & 0 & r^{-1} & \mu^{-1} \\ \rho g r^{-1}-2\gamma r^{-2} & -\lambda\sigma^{-1}r^{-1} & -2\mu r^{-2}+(\gamma+\mu)l(l+1)r^{-2} & -3r^{-1} \end{pmatrix}$$

$$\mu = \mu(s)$$

$$\lambda = \lambda(s) = \kappa(s) - \frac{2}{3}\mu(s)$$

$$\sigma = \lambda(s) + 2\,\mu(s)$$

$$\gamma = \lambda(s) + \mu(s) - \lambda^2(s)\,\sigma^{-1}$$

Toroidal motion

$$\mathbf{A}_T(r,l) = \begin{pmatrix} r^{-1} & \mu^{-1}(s) \\ r^{-2}\mu(s)(l-1)(l+2) & -3r^{-1} \end{pmatrix}$$

Stability of spheroidal mode solution

Method of second order minors

- Two displacement-stress vectors $\mathbf{y}_1(r)$ and $\mathbf{y}_2(r)$ (assuming fixed l and s) are propagated upward from the starting radius of integration.
- Second-order minors:

$$m_1(r) = m_{12} = \begin{vmatrix} y_{11} & y_{21} \\ y_{12} & y_{22} \end{vmatrix}, \qquad m_2(r) = m_{13} = \begin{vmatrix} y_{11} & y_{21} \\ y_{13} & y_{23} \end{vmatrix}$$

$$m_3(r) = m_{14} = \begin{vmatrix} y_{11} & y_{21} \\ y_{14} & y_{24} \end{vmatrix}, \qquad m_4(r) = m_{23} = \begin{vmatrix} y_{12} & y_{22} \\ y_{13} & y_{23} \end{vmatrix}$$

$$m_5(r) = m_{24} = \begin{vmatrix} y_{12} & y_{22} \\ y_{14} & y_{24} \end{vmatrix}, \qquad m_6(r) = m_{34} = \begin{vmatrix} y_{13} & y_{23} \\ y_{14} & y_{24} \end{vmatrix},$$

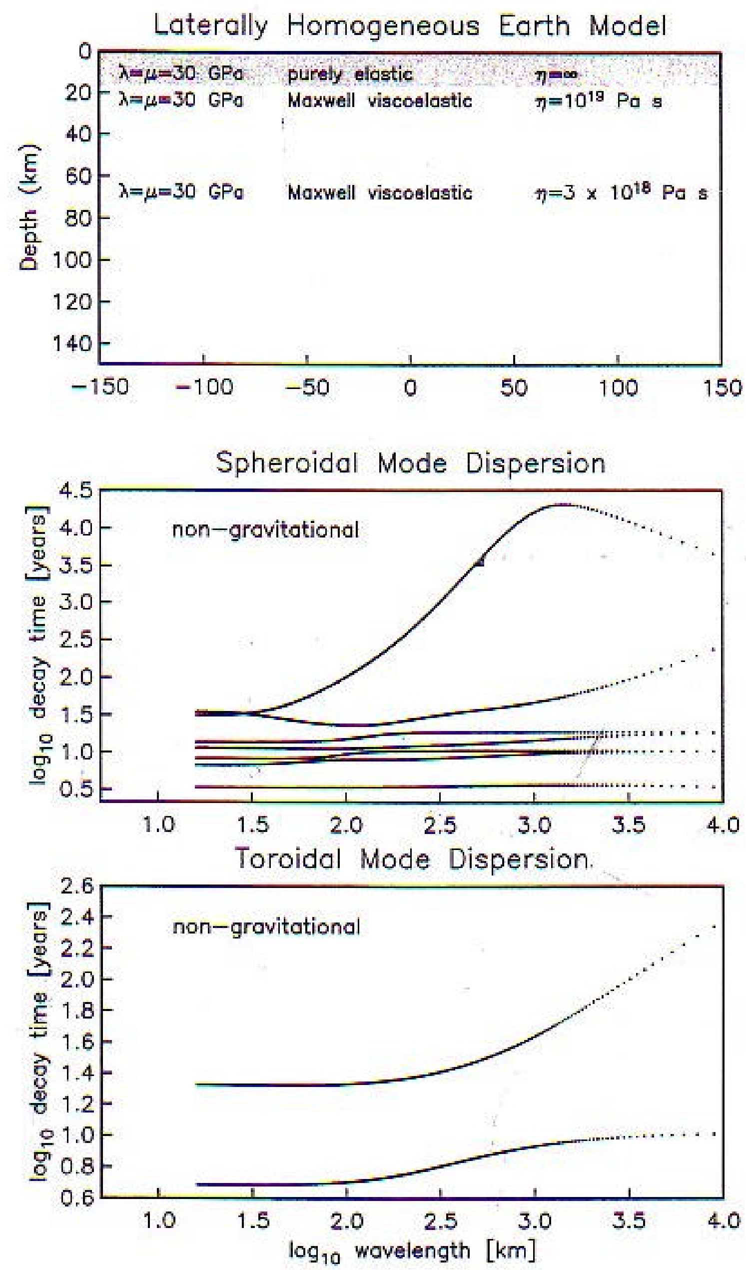
$$m_6 = -\frac{1}{l(l+1)} m_1$$

- Differential equation for minors:

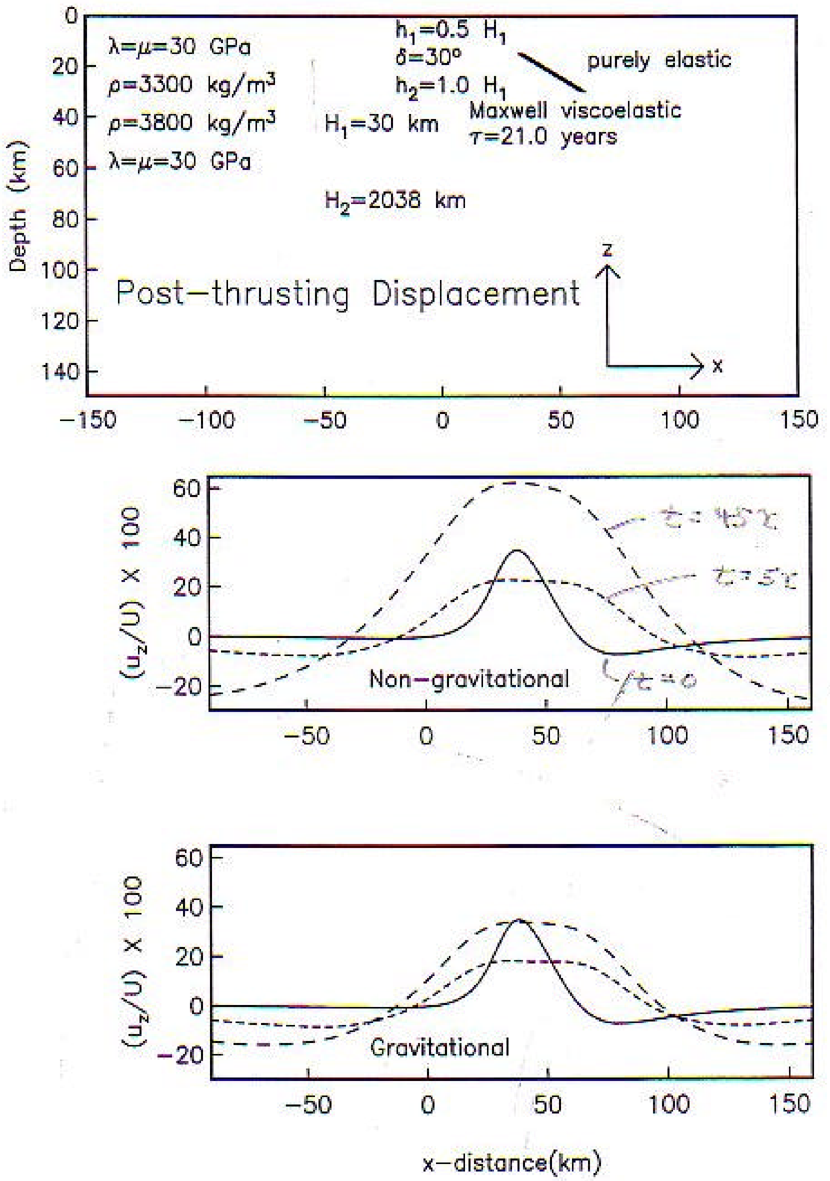
$$\frac{dm_{jk}(r)}{dr} = \sum_k (\mathbf{A}_S)_{jl}\, m_{lk} + (\mathbf{A}_S)_{kl}\, m_{jl}$$

- Surface boundary condition:

$$m_5(a) = m_{24}(a) = 0$$ ← stable


Laterally Homogeneous Earth Model
λ=μ=30 GPa purely elastic η=∞
λ=μ=30 GPa Maxwell viscoelastic η=10^19 Pa s
λ=μ=30 GPa Maxwell viscoelastic η=3 x 10^18 Pa s
Depth (km)
Spheroidal Mode Dispersion
non-gravitational
log10 decay time [years]
Toroidal Mode Dispersion
non-gravitational
log10 decay time [years]
log10 wavelength [km]

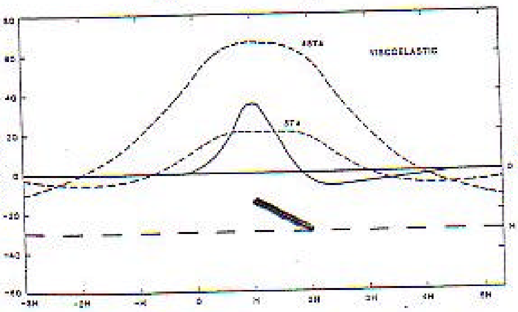
VISCO1D

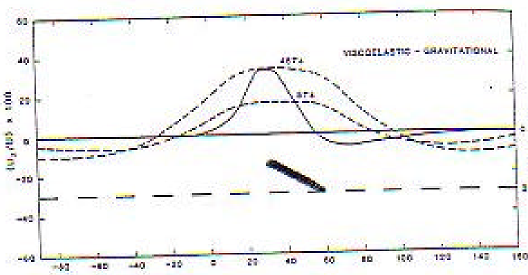

λ=μ=30 GPa
ρ=3300 kg/m³
ρ=3800 kg/m³
λ=μ=30 GPa
h₁=0.5 H₁
δ=30°
h₂=1.0 H₁
purely elastic
Maxwell viscoelastic
τ=21.0 years
H₁=30 km
H₂=2038 km
Post–thrusting Displacement
z
x
Depth (km)
0
20
40
60
80
100
120
140
−150
−100
−50
0
50
100
150
(u_z/U) X 100
60
40
20
0
−20
t = 45τ
t = 5τ
t = 0
Non–gravitational
Gravitational
x–distance(km)

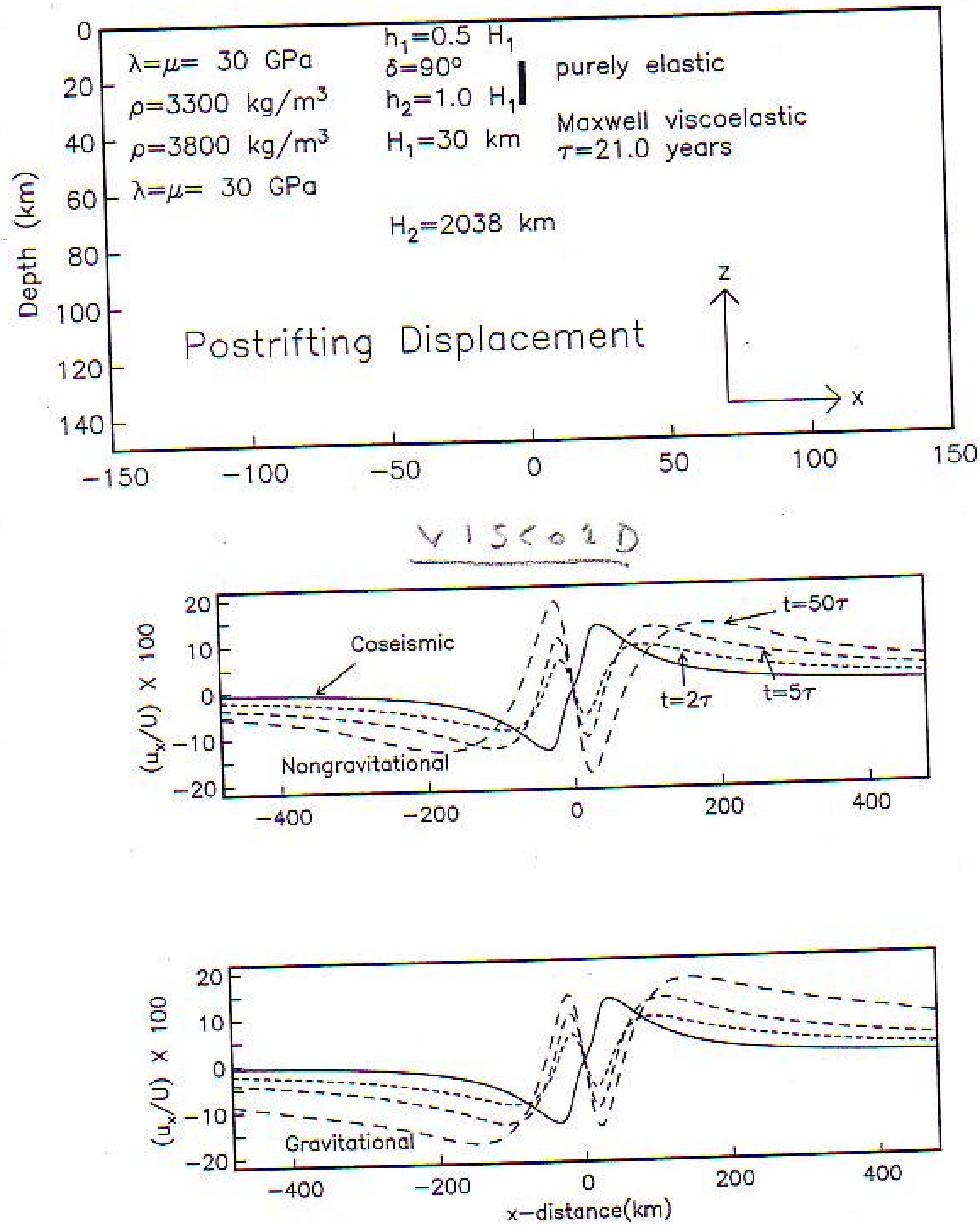
Assumes relaxing shear modulus μ
nonrelaxing Lamé parameter λ
($\lambda = K - \frac{2}{3}\mu$, where K is the bulk modulus)

Post-rifting displacement

Rundle's code



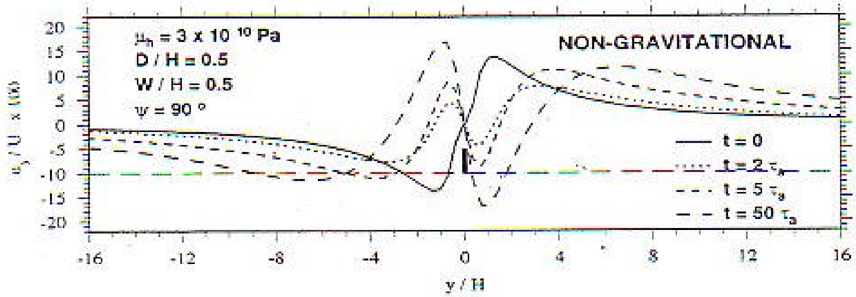


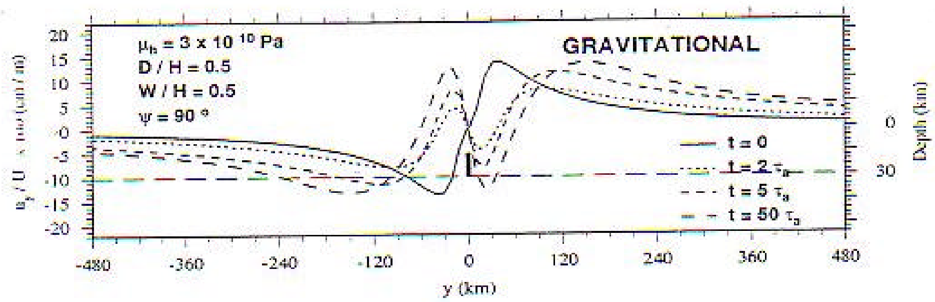


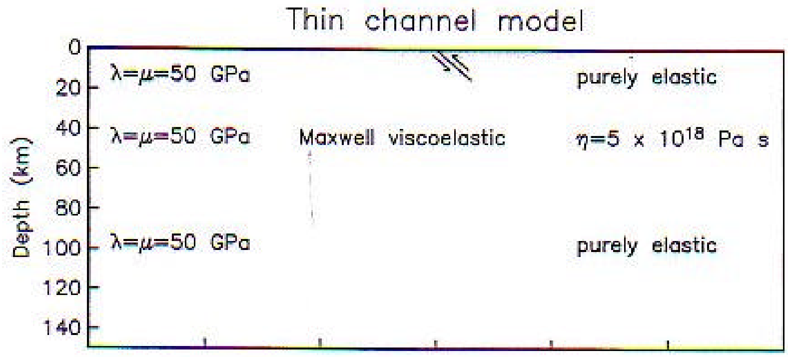
λ=μ= 30 GPa
ρ=3300 kg/m³
ρ=3800 kg/m³
λ=μ= 30 GPa
h1=0.5 H1
δ=90°
h2=1.0 H1
H1=30 km
H2=2038 km
purely elastic
Maxwell viscoelastic
τ=21.0 years
Postrifting Displacement
Depth (km)
z
x
VISCO2D
Coseismic
Nongravitational
t=50τ
t=2τ
t=5τ
(ux/U) X 100
Gravitational
x−distance(km)

Rundle's code

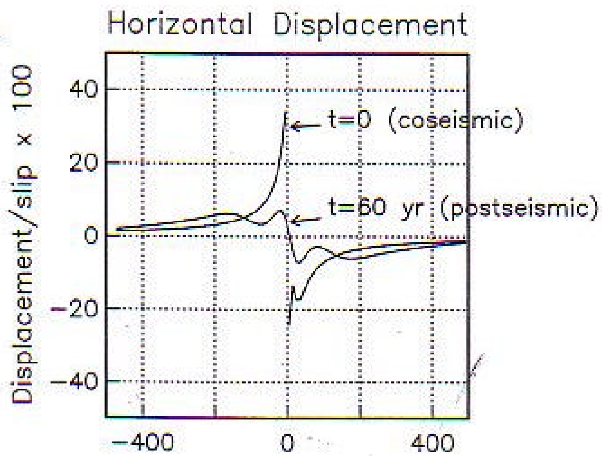
 (1995 JGR)

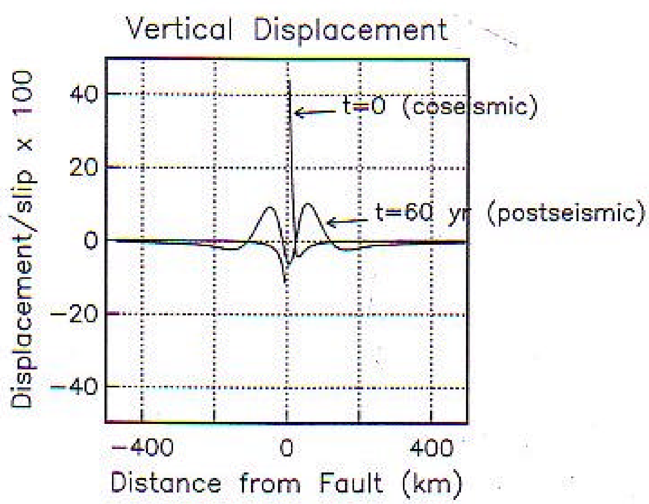





Thin channel model
λ=μ=50 GPa
purely elastic
λ=μ=50 GPa
Maxwell viscoelastic
η=5 x 10^18 Pa s
λ=μ=50 GPa
purely elastic
Depth (km)
0
20
40
60
80
100
120
140

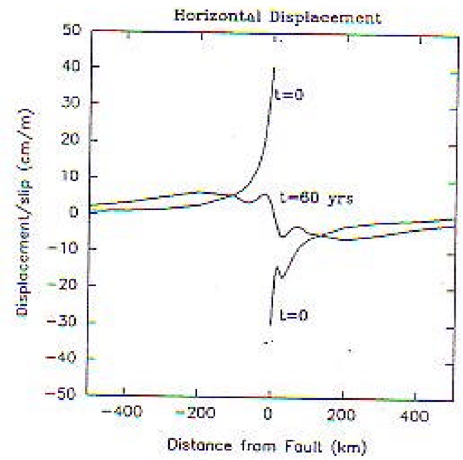
VISCO1D

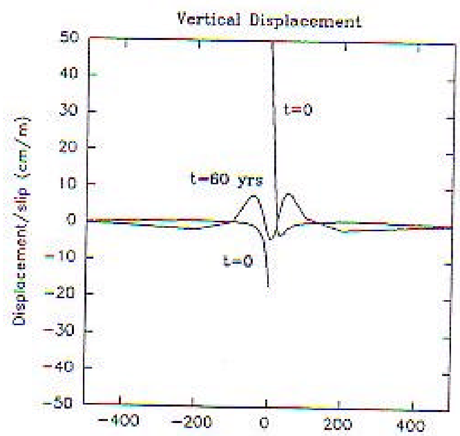

Horizontal Displacement
Displacement/slip x 100
t=0 (coseismic)
t=60 yr (postseismic)
40
20
0
−20
−40
−400
0
400


Vertical Displacement
Displacement/slip x 100
t=0 (coseismic)
t=60 yr (postseismic)
40
20
0
−20
−40
−400
0
400
Distance from Fault (km)

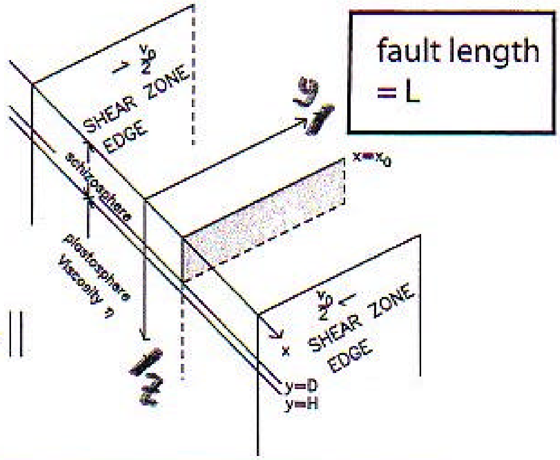
 (Cohen, 1992)

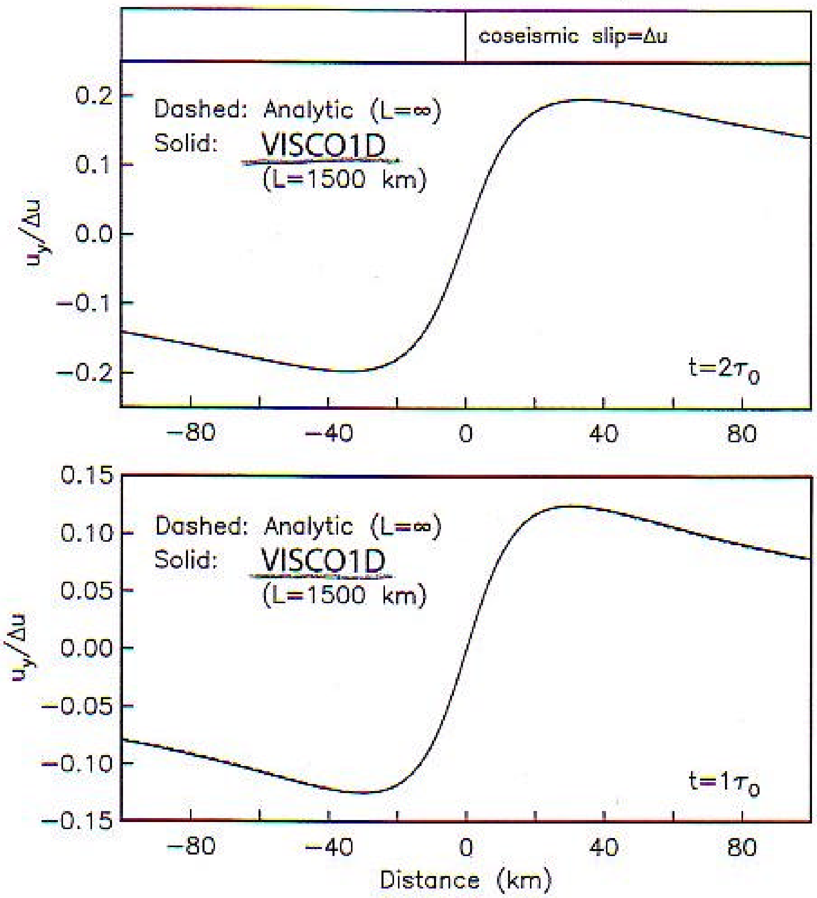
TECTON





Postseismic fault-|| displacement





VISCO3D

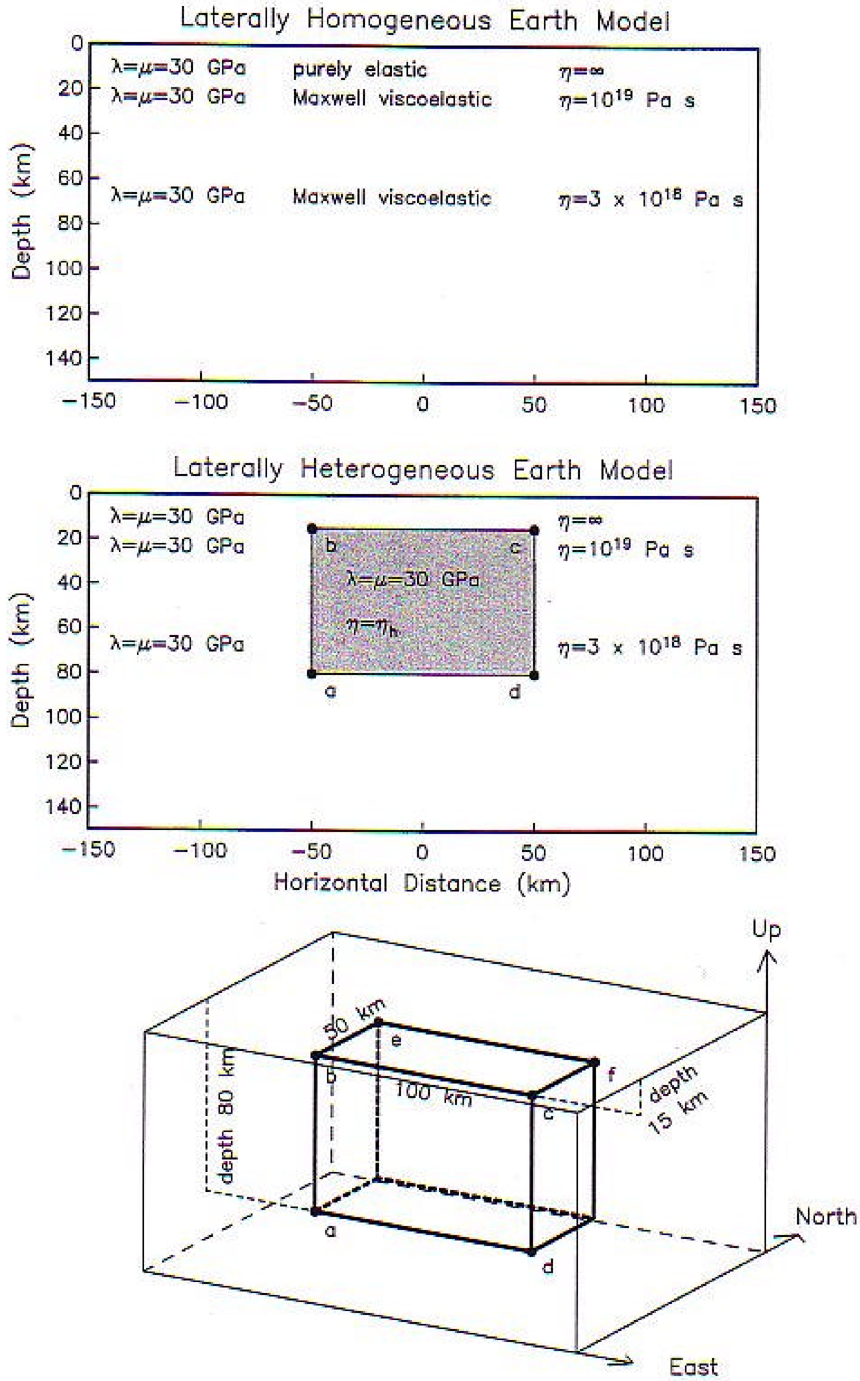
- Computes postseismic response to impulsive seismic sources on aspherical viscoelastic model

- Based on perturbation theory for aspherical structure with respect to a laterally homogeneous model

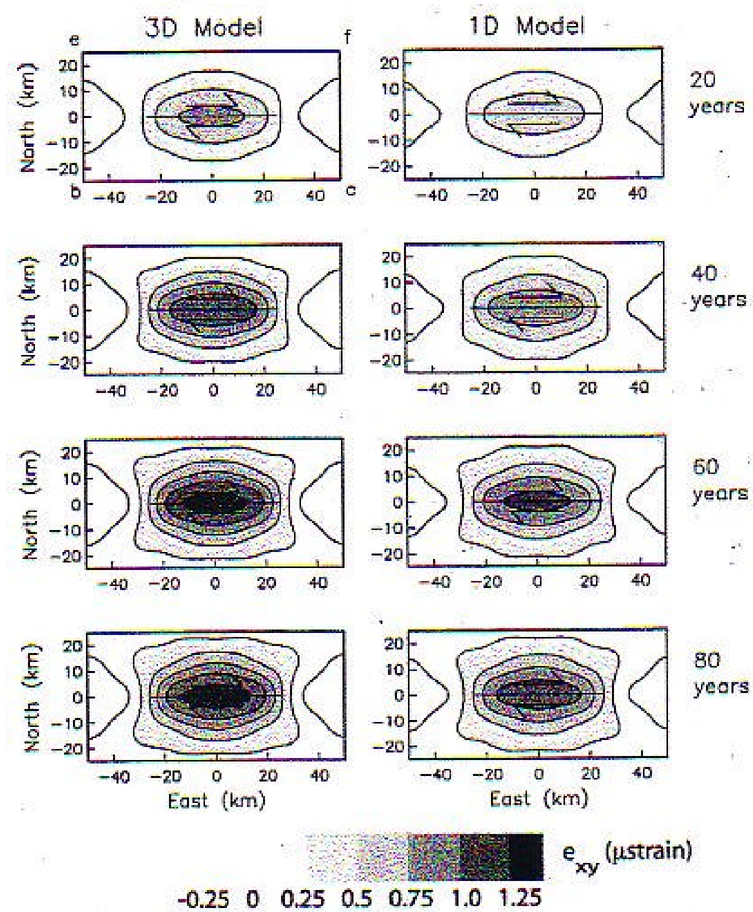
- Coupled static + viscoelastic normal modes

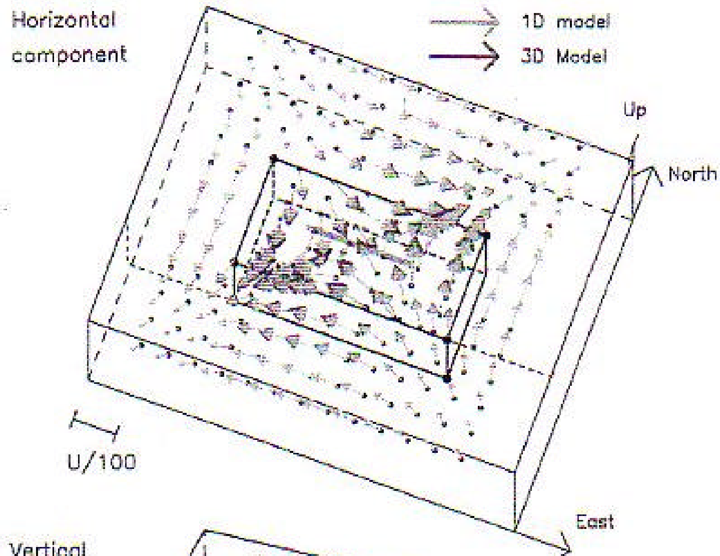
- Numerical effort proportional to volume of aspherical region

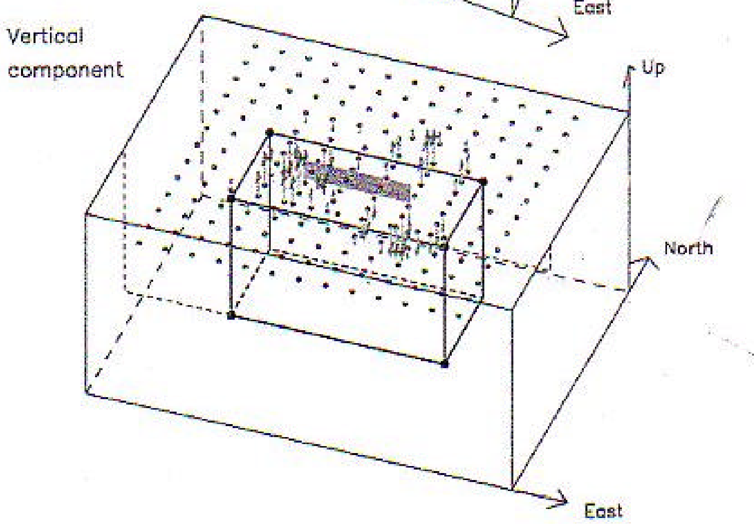
- Accounts for effects of linear rheology and gravitational acceleration (g–effects, but not G–effects)



Laterally Homogeneous Earth Model
λ=μ=30 GPa
purely elastic
η=∞
λ=μ=30 GPa
Maxwell viscoelastic
η=10^19 Pa s
λ=μ=30 GPa
Maxwell viscoelastic
η=3 x 10^18 Pa s
Depth (km)
0
20
40
60
80
100
120
140
−150
−100
−50
0
50
100
150
Laterally Heterogeneous Earth Model
λ=μ=30 GPa
λ=μ=30 GPa
λ=μ=30 GPa
η=∞
η=10^19 Pa s
η=3 x 10^18 Pa s
λ=μ=30 GPa
η=η_h
a
b
c
d
Depth (km)
Horizontal Distance (km)
Up
North
East
50 km
100 km
depth 80 km
depth 15 km
e
f


3D Model
1D Model
e
f
b
c
North (km)
East (km)
20 years
40 years
60 years
80 years
e_{xy} (μstrain)
-0.25 0 0.25 0.5 0.75 1.0 1.25


Horizontal
component
1D model
3D Model
Up
North
U/100
East


Vertical
component
Up
North
East