12.010 Homework #4 **Due Thursday November 18, 2010**

Submissions: For this homework, you should submit a Mathematica NoteBook. Your submissions should include
(1) A description of how you solved the problem. This description can be in the NoteBook;
(2) The specific output requested for each problem which can be imbedded in the NoteBook (save NoteBook once solutions are complete); and
(3) The Mathematica code that you wrote in a NoteBook. (I run your codes.) If your code has "strange" behavior such as crashing, infinite loops, or excessively long run times (more than a few seconds), let me know in your submissions.

NOTE: You probably should re-start Mathematica or its Kernel and run your NoteBook before saving the submitted copy. (This ensures that you are not using variables and expressions from earlier in your session).

I strongly recommend that you start early on these problems and send questions or come to see me as soon as you encounter problems. You can email tah@mit.edu to set a time for an appointment. Office hours are Tuesday 1:00-2:00 pm.

Question (1): (25-points) Write a program that reads a file containing text, counts the number of characters (letters a-z) and words in the text, and output the text with capital letters at the beginning of each sentence. The text below is contained in the file Q1_text.txt.

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we take as a self evident foundational principle that the set of effects
to be considered as contributing to local station displacements and the
conventional models to be applied for their compensation should be
guided by rational and well considered bases, and should not be developed
haphazardly or randomly.  for historical reasons and general consistency,
it might be prudent to retain some past practices even if they are not
fully consistent with the adopted principles; but future expansions
should be determined by specified rules.  this position paper proposes
such a set of guidelines and rationales for IERS conventions updates.
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Hints:
(1) Mathematica has many built in String Functions and you should check to see which ones are available and can be used for this problem.

Question (2): (25-points) Write a program to accurately sum a large number of floating point numbers. The floating point numbers to be summed should be constructed as random numbers of the form
Value = mantissa*$10^{exponent}$
Where mantissa and exponent are generated as random numbers between 0-1 and exponent as random number between 0 and max_exponent. These numbers should be generated with 7 significant digits of accuracy.
max_exponent should be a user specified value (typical value would be 10) and the

number of values to be summed should also be user specified (typical value 10000). The sum should be made using an accuracy of 7-significant digits and your results can be compared to a summation performed with high-accuracy to see how accurate your result is. The program should try to achieve the maximum summation accuracy and output the accuracy of the low accuracy sum. Your submission should explain how your algorithm achieves its accuracy using only low precision variables and the different methods you considered. Can a formula be found that allows a prediction of the accuracy of the algorithm?
Use the SetAccuracy function in Mathematica to perform the 7-significant digit calculation. Compare result with higher accuracy calculation.

Question 3: **(50-points)** Solution to the Lorenz Strange Attractor problem. Implement a solution to the following differential equations that define the Lorenz Strange Attractor problem:

$$x' = \sigma(y - x)$$

$$y' = -xz + Rx - y$$

$$z' = xy - Bz$$

where *x', y'* and *z'* are *dx/dt*; *dy/dt* and *dz/dt* where *t* is time; σ is Prandtl numbers, *R* is the ratio of Rayleigh numbers and *B* is geometric factor.
Your program should take as input values for σ, *R* and *B*; the initial values of *x, y*, and *z*; the length of time for the solution and the accuracy of the solution. The accuracy can be specified as relative or absolute error. (Absolute error is the absolute difference between your solution and the real solution; relative error is the absolute error divided by the typical size of the variable. The relative error is not dependent on the units of *x,y,z*).
Your solution should explain how you solved the differential equations and the method to used to achieve the desired accuracy of the solution.
(a) To test your program: Output at 1 second intervals, the solution using:
$\sigma = 10$; $R= 40$; and $B=8/3$;
initial values of $x=0$, $y=-10$, and $z=0$;
Integration should be for 20-seconds. The accuracy should be errors of order 10^{-4}. Take advantage of Mathematica ordinary differential equation solvers for this problem.
(b) Generate a 3-D plot of the you result, and time series plots of x, y and z.