

# 12.215 Modern Navigation

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## Review of last Class

- GPS measurements
  - Tracking methods used in GPS (“codeless” tracking)
  - Basic geometry of orbits (discuss more later)
  - Specific details of the GPS signal structure

## Today's class

- GPS measurements
  - Basics of pseudorange measurements
  - Phase measurements (allow millimeter level position with GPS and cm in real-time)
  - Examine some GPS data.
- Positioning modes
- Dilution of precision numbers

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## Basic measurement types

- Substituting into the equation of the pseudorange yields

$$P_k^p = [(\tau_k - \tau^p) + (\Delta t_k - \Delta t^p)] \cdot c$$

$$P_k^p = \rho_k^p + (\Delta t_k - \Delta t^p) \cdot c + \underbrace{I_k^p}_{\text{Ionospheric delay}} + \underbrace{A_k^p}_{\text{Atmospheric delay}}$$

- $\rho_k^p$  is true range, and the ionospheric and atmospheric terms are introduced because the propagation velocity is not  $c$ .

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## Basic measurement types

- The equation for the pseudorange uses the true range and corrections applied for propagation delays because the propagation velocity is not the in-vacuum value,  $c$ ,  $2.99792458 \times 10^8$  m/s
- To convert times to distance  $c$  is used and then corrections applied for the actual velocity not equaling  $c$ . (Discussed in later lectures)
- The true range is related to the positions of the ground receiver and satellite.
- We also need to account for noise in measurements

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## Pseudorange noise

- Pseudorange noise (random and not so random errors in measurements) contributions:
  - **Correlation function width:** The width of the correlation is inversely proportional to the bandwidth of the signal. Therefore the 1MHz bandwidth of C/A produces a peak 1  $\mu$ sec wide (300m) compared to the P(Y) code 10MHz bandwidth which produces 0.1  $\mu$ sec peak (30 m)  
Rough rule is that peak of correlation function can be determined to 1% of width (with care). Therefore 3 m for C/A code and 0.3 m for P(Y) code.

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## Pseudorange noise

- More noise sources
  - **Thermal noise:** Effects of other random radio noise in the GPS bands  
Black body radiation:  $I=2kT/\lambda^2$  where  $I$  is the specific intensity in, for example, watts/(m<sup>2</sup>Hz ster),  $k$  is Boltzman's constant,  $1.380 \times 10^{-23}$  watts/Hz/K and  $\lambda$  is wavelength.  
Depends on area of antenna, area of sky seen (ster=steradians), temperature  $T$  (Kelvin) and frequency. Since P(Y) code has narrower bandwidth, tracking it in theory has 10 times less thermal noise power (cut by factor of 2 because less transmission power)  
Thermal noise is general smallest effect
  - **Multipath:** Reflected signals (discussed later)

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## Pseudorange noise

- The main noise sources are related to reflected signals and tracking approximations.
- High quality receiver: noise about 10 cm
- Low cost receiver (\$200): noise is a few meters (depends on surroundings and antenna)
- In general: C/A code pseudoranges are of similar quality to P(Y) code ranges. C/A can use narrowband tracking which reduces amount of thermal noise
- Precise positioning (P-) code is not really the case.

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## Phase measurements

- Carrier phase measurements are similar to pseudorange in that they are the difference in phase between the transmitting and receiving oscillators. Integration of the oscillator frequency gives the clock time.
- Basic notion in carrier phase:  $\phi = f\Delta t$  where  $\phi$  is phase and  $f$  is frequency
- “Big” problem is know the number of cycles in the phase measurements

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## Phase measurements

$$\phi_k^p(t_r) = \phi_k(t_r) - \phi_r^p(t_r) + N_k^p(1)$$

- The carrier phase is the difference between phase of receiver oscillator and signal received plus the number of cycles at the initial start of tracking
- The received phase is related to the transmitted phase and propagation time by

$$\phi_r^p(t_r) = \phi_t^p(t_t) = \phi_t^p(t_r - \rho_k^p/c) = \phi_t^p(t_r) - \dot{\phi}^p(t_r) \cdot \rho_k^p/c$$

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## Phase measurements

- The rate of change of phase is frequency. Notice that the phase difference changes as  $\rho/c$  changes. If clocks perfect and nothing moving then would be constant.
- Subtle effects in phase equation
  - Phase received at time  $t$  = phase transmitted at  $t-\tau$  (riding the wave)
  - Transmitter phase referred to ground time (used later). Also possible to formulate as transmit time.

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## Phase measurements

- When phase is used it is converted to distance using the standard L1 and L2 frequencies and vacuum speed of light.
- Clock terms are introduced to account for difference between true frequencies and nominal frequencies. As with range ionospheric and atmospheric delays account for propagation velocity

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## Precision of phase measurements

- Nominally phase can be measured to 1% of wavelength (~2mm L1 and ~2.4 mm L2)
- Again effected by multipath, ionospheric delays (~30m), atmospheric delays (3-30m).
- Since phase is more precise than range, more effects need to be carefully accounted for with phase.
- Precise and consistent definition of time of events is one the most critical areas
- In general, phase can be treated like range measurement with unknown offset due to cycles and offsets of oscillator phases.

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## GPS data

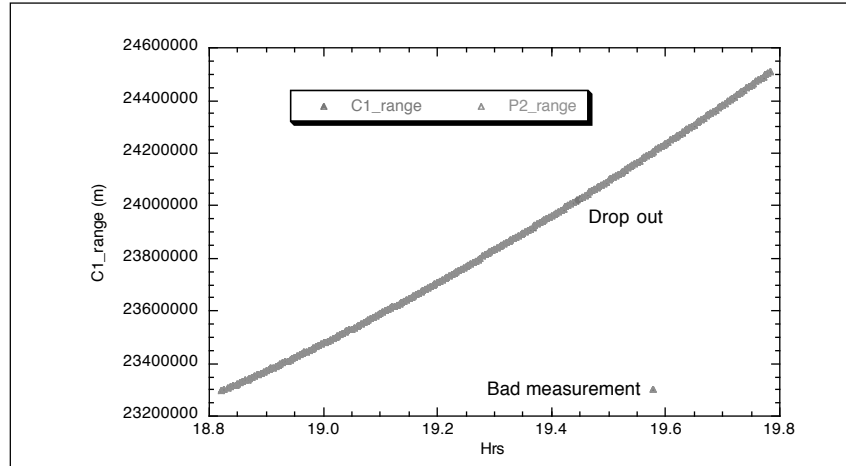
- Next set of plots will look at the GPS data
- Examples for one satellite over about 1 hour interval:
  - Raw range data
  - Raw phase data
  - Differences between data

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## Raw range data

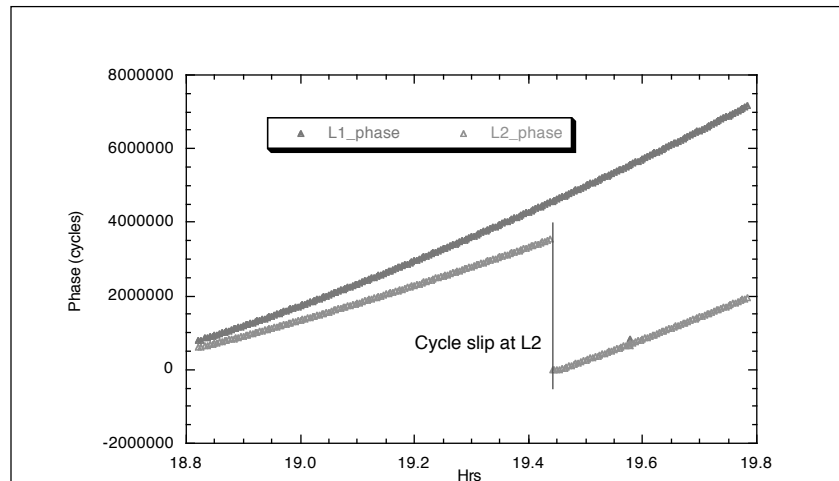


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## Raw phase data (Note: sign)



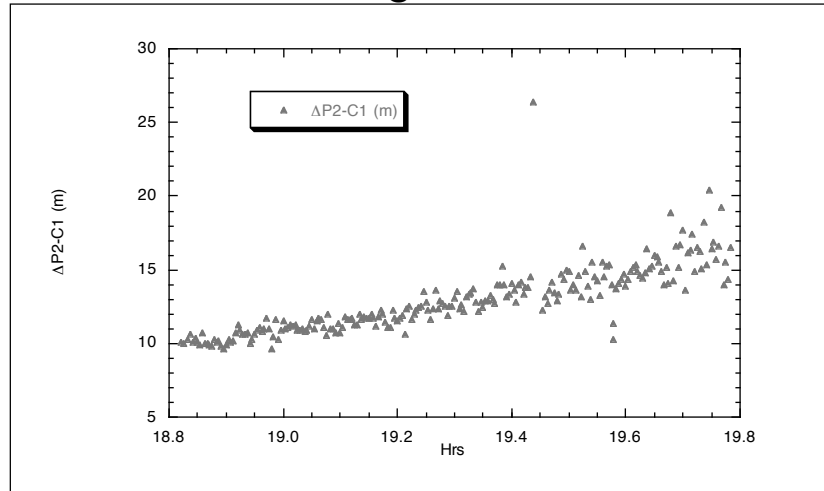
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## L2-L1 range differences

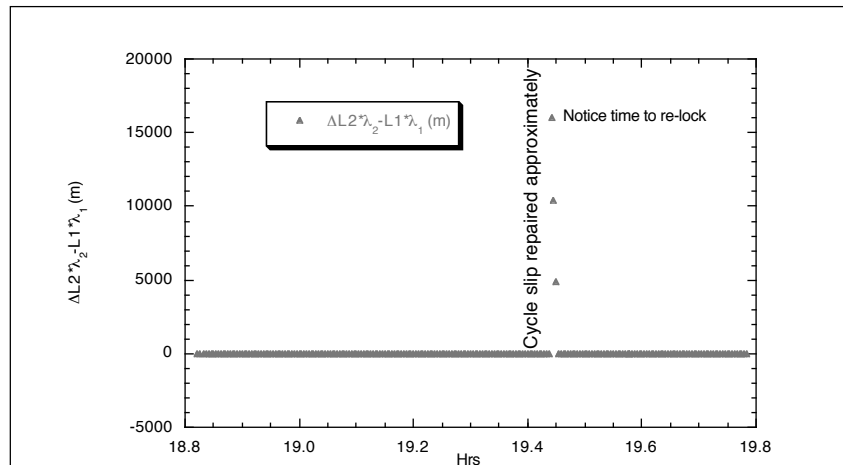


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## L2-L1 phase differences

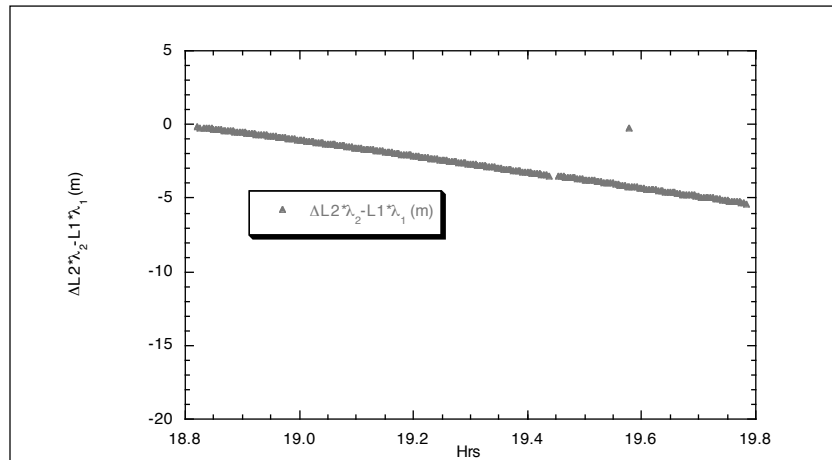


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## Zoomed L2-L1 phase



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## Basic GPS analysis methods

- The issue that must be addressed in GPS processing if the unknown changes in the receiver and satellite clocks.
- For low precision positioning (tens of meters) the satellite clocks are assumed known and given by the broadcast ephemeris.
  - Receiver clock can be estimated along with 3-D positions if 4 or more satellites are visible.
  - Alternatively, the positions can be estimated from the difference between the measurements to a satellite and a reference satellite.

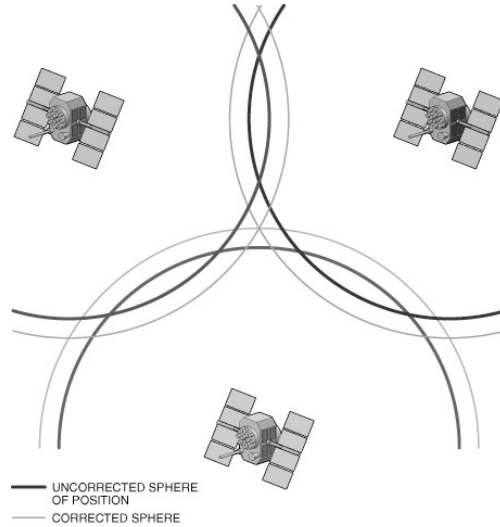
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## Basic positioning

- Diagram below a 2-D example of effects of receiver clock.
- Notice: measured thick lines to not meet; thin lines after applying a constant offset, meet at one point.



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## Precise positioning

- For better than tens of meters positioning, better information about satellite clocks is needed
- Differential GPS (DGPS) uses the pseudorange measurements from a known location to effectively estimate the error in the satellite clocks (and some other effects as well).
- By applying these clock corrections to the pseudorange measurements at a site of unknown coordinates, the errors due to satellite clocks can be largely removed.
- The clock corrections can be transmitted by radio (RTCM model) for nearby stations (US Coast Guard system), or from satellite (Wide Area Augmentation system, WAIS).
- In WAIS, data from many known locations is averaged to reduce noise.

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## Representation of accuracy

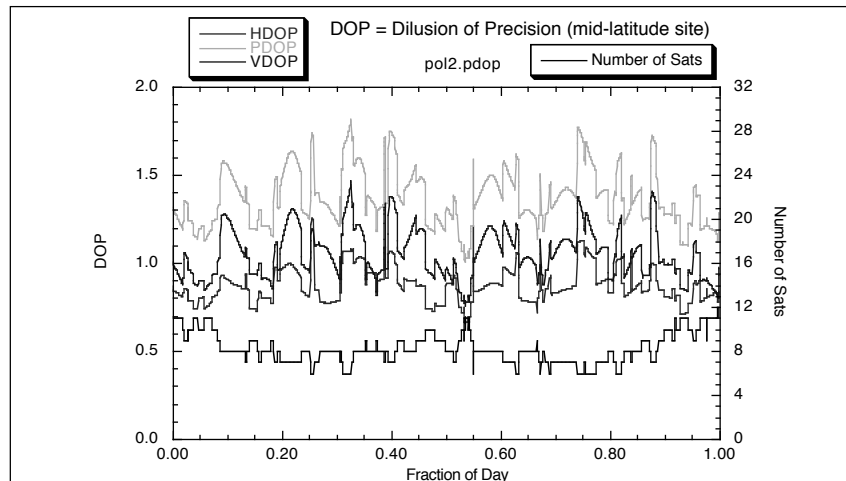
- In GPS applications (especially real-time applications in which positions are determined “instantaneously”), precision is represented by Dilution of Precision (DOP) values.
- DOPs are the ratio of the position precision to range noise precision.
  - PDOP: Over all 3-D position precision
  - HDOP: Horizontal position precision
  - VDOP: Vertical position precision
- Example on next page is for mid-latitude site.

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## Mid-Latitude DOP



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