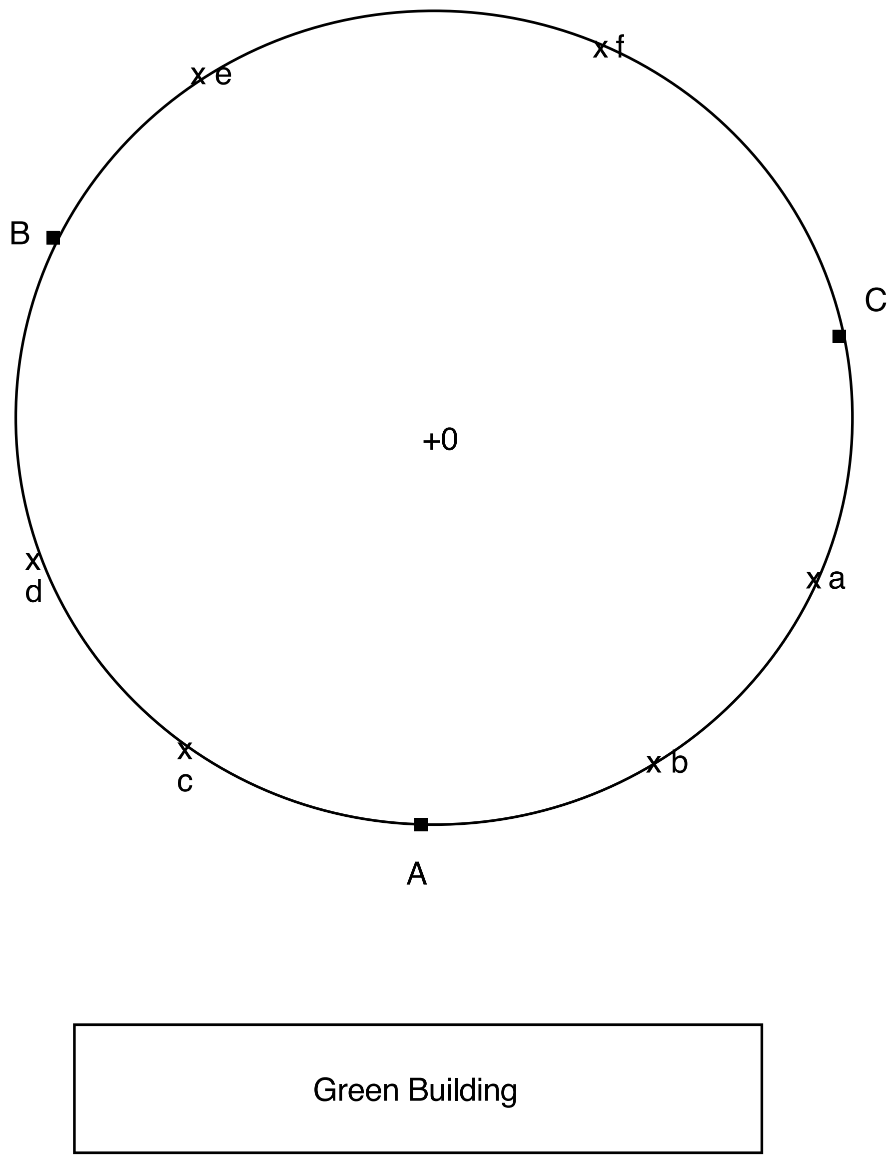
12S56 Circle data **Collected 11/03/2009**



Data

AT	**TO**	**Angles (deg)**	**Distance (m)**
A	B	0.0001	34.371
	C	67.5996	34.359
B	C	0.0001	38.233
	A	56.1876	34.371
C	A	0.0001	38.362
	B	56.2187	38.233
	a	315.2607	8.027

b	338.1861	23.402
c	20.5806	40.271
d	43.8662	40.753
e	70.5921	33.013
f	93.4452	20.656
O	33.8481	20.702

Solution:
Solution adjustment. The first step in the analysis is to make the angles consistent (i.e., sum to 180 degrees). These adjustments are usually made by distributing the "misclose" (the difference from 180 deg), into each angle inversely proportional to the line lengths. In our case the lengths are all about the same length so we subtract 0.002 deg to each angle. (This corresponds to mis-pointing by ~1.0mm over the 33-39 meter distances). The distance measurements all agree in the forward and back directions except for one 1 mm difference. The first measurement was adopted.
(a) Using the geometry from the figure above at site 00, we can write two equations for the radius:

$$R\cos\alpha_2 = d_2/2$$
$$R\cos\alpha_1 = d_1/2 \quad \text{where } \alpha_1 + \alpha_2 = \alpha$$

The division of these two equations results in the R being canceled and using the expansion of $\cos(\alpha - \alpha_1) = \cos\alpha\cos\alpha_1 + \sin\alpha\sin\alpha_1$ we can write

$$\frac{\cos\alpha\cos\alpha_1 + \sin\alpha\sin\alpha_1}{\cos\alpha_1} = \frac{d_2}{d_1}$$

By expansion, this equation reduces to:

$$\tan\alpha_1 = (\frac{d_2}{d_1} - \cos\alpha)/\sin\alpha$$

Using the estimate of α_1, we can then solve for the radius R.
For each corner point the results are:

$$\tan\alpha_1 = (\frac{d_2}{d_1} - \cos\alpha)/\sin\alpha \Rightarrow \alpha_1 = 33.8125\,\text{deg} \Rightarrow R = 20.677\,m$$

$$\tan\beta_1 = (\frac{d_0}{d_2} - \cos\beta)/\sin\beta \Rightarrow \beta_1 = 33.7836\,\text{deg} \Rightarrow R = 20.677\,m$$

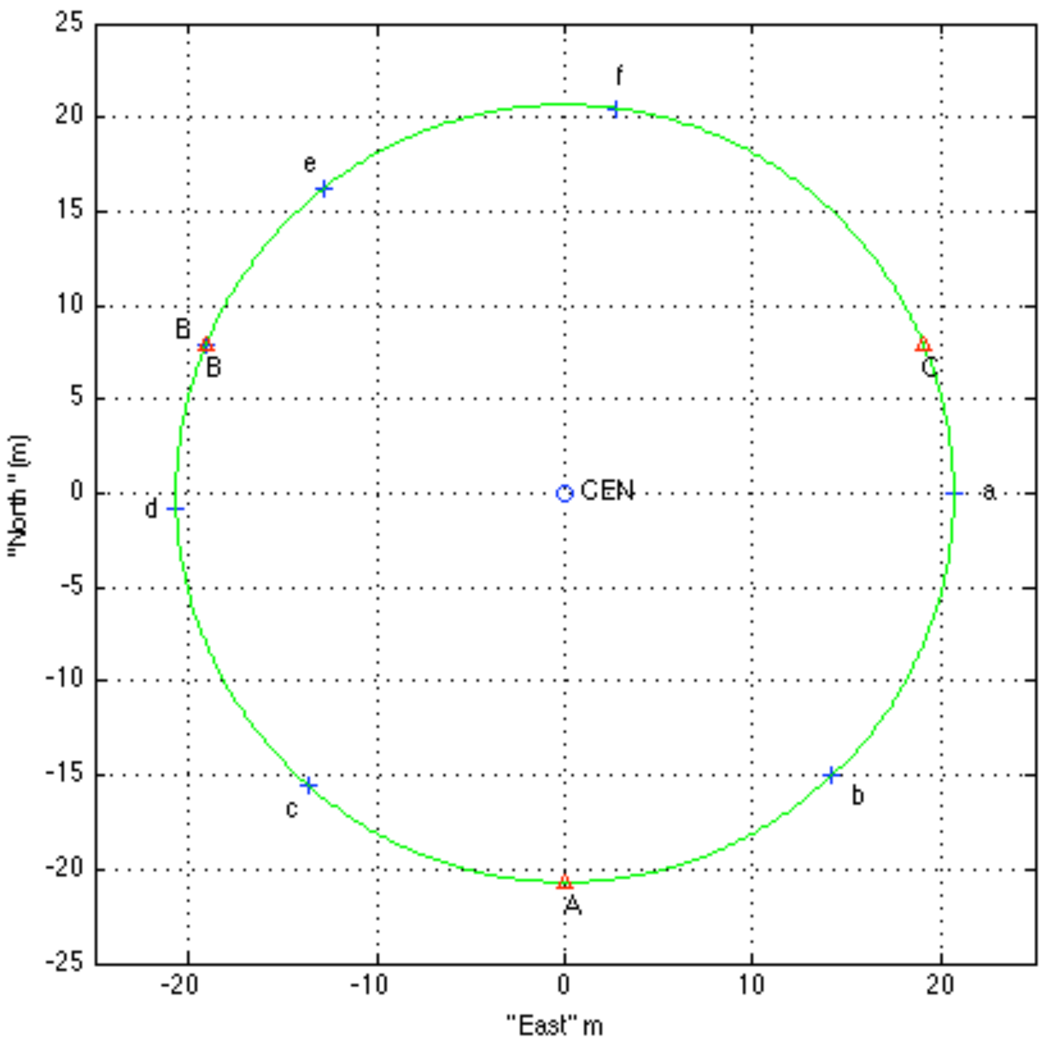
$$\tan\gamma_1 = (\frac{d_1}{d_0} - \cos\gamma)/\sin\gamma \Rightarrow \gamma_1 = 22.4044\,\text{deg} \Rightarrow R = 20.677\,m$$

(b) To find the radius to each of the intermediate points, we use the data from site C. The cosine rule is used to solve for r and the sine rule to solve for ψ. To solve these equations we use:

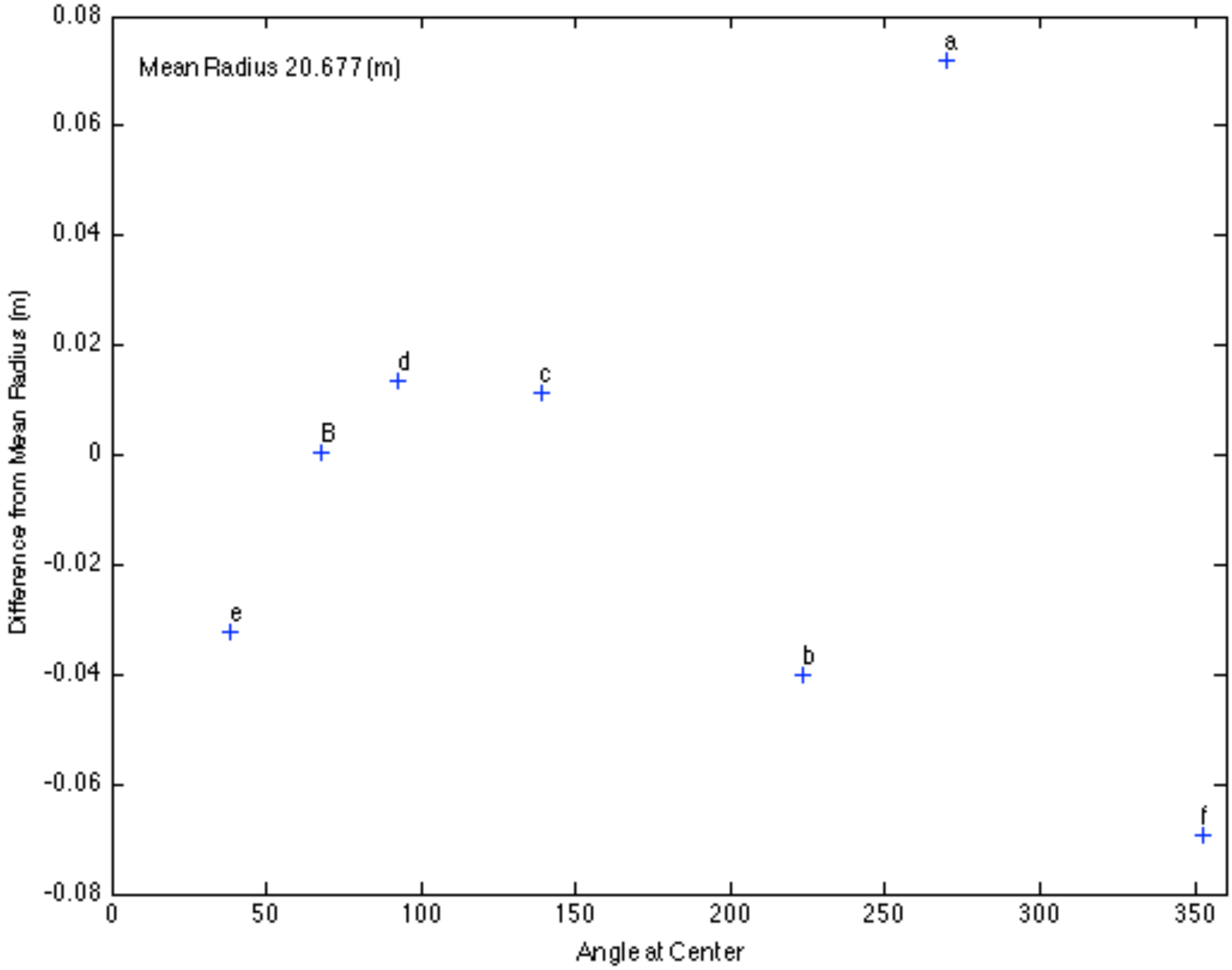
$$\tan\alpha'_1 = (\frac{d}{d_1} - \cos\alpha')/\sin\alpha' \Rightarrow r = d'/(2\cos\alpha'_1)$$
$$\psi = 2[90 - (\alpha' - \alpha'_1)]$$

(c) The position of the sprinkler at the center (CEN) and computed by geometry. If the spigot had been exactly at the center, the distance to it would have been 20.677 m (compared to the measured value of 20.702 m). The difference in position places the spigot 0.028 m from the center at ψ =-27 deg.

The total results are shown in the figure below. ("South" is the direction from the center of the circle to point A, "East" at right angles to this direction.



The residuals to the mean radius and a function of the angle at the center are in the figure below:



This project was solved using Matlab code Proj_3_09.m. The output of the code (in addition to the figures above is:

```
12S56 Project Number 3
Sum of angles in triangle is 180.0057 deg, adding -0.0019
to each angle
-------12S56 2009------------------------
Results for each angle/distance pair
Alpha    1  33.8125  Radius 1 20.677
Beta     1  33.7836  Radius 2 20.677
Gamma    1  22.4044  Radius 2 20.677
Mean radius 20.677
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 Point      Radius     Drad  Angle
    B     20.677     0.000  67.5639
    a     20.749     0.072  270.0690
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b	20.637	-0.040	223.2940
c	20.689	0.011	138.9328
d	20.691	0.014	92.3788
e	20.645	-0.032	38.5476
f	20.608	-0.069	352.5300

Sprinkler Position 0.028 (m) at -27.39 deg