

12S56 GPS

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MW 11:00-12:30 Room 54-322
<http://geoweb.mit.edu/~tah/12S56>**

Today's Seminar

- Electronic Distance Measurement (EDM)
- History
- Methods:
 - Theory: Propagating electromagnetic signals
 - Timing signal delays
 - Use of phase measurements
 - Application areas (other than GPS)

How to measure distances

- Astronomical measurements tell us position relative to gravity field. If we want to measure distances directly, how do we do this?
- **Rulers:** Steel tapes -- good to ~100 meters
- **Triangulation:** Dates from ~1600 and Snell (refraction of light Snell).
- **Trilateration:** Uses distance measurements measured remotely (i.e., via a propagating signal: Electromagnetic most common, sound is also possible but not accurate)

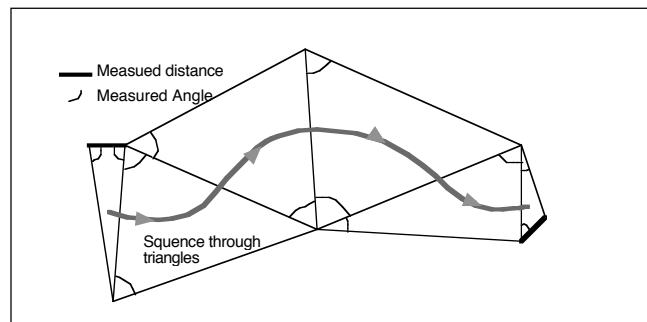
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Basic Triangulation

- Angles in figure measured and with 1 distance measured, size and shape of figure can be determined



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History of EDM

- Development of this type of technology started during World War II with the development of RADAR (Radio Detection and Ranging)
- Radars returned the distance to an object (and later versions the speed of the object through the Doppler shift) by timing the length of time the from the transmission of a pulse to its return. Accuracy was set by timing resolution ($1\mu\text{sec}=300\text{meters}$)
- In 1949, Dr. Erik Bergstrand of Sweden introduced the Geodimeter (Geodetic Distance Measurement) that used light (550 nm wavelength) to measure geodetic quality distances (instrument weighed 100kg)

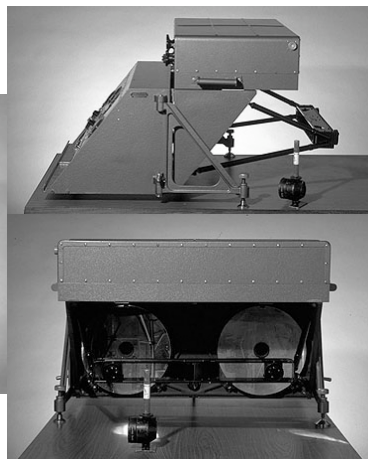
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Geodimeter

- First units circa 1959 (50 kg each for measurement unit and optics)



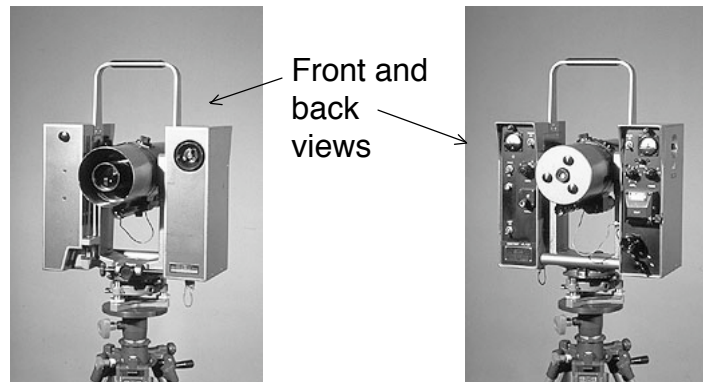
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Later model Geodimeter

- Example of a latter model Geodimeter (circa 1966)



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History of EDM

- Distance range was about 10km during daylight and 25km at night.
- Greater range during daytime was achieved by using radio waves, and in Dr. T. L. Wadley, South Africa introduced the Telluometer in 1957.
- Instrument used X-band radio waves ($\sim 10\text{GHz}$)
- Receive and transmit ends looked similar (receiver actually re-transmitted the signal) (The Geodimeter used one or more corner cube reflectors.)
- Distances up to 50 km could be measured in daylight with this instrument and later models.

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Telluometer

- Example of circa 1962 model.



Back and front of
instrument (9 kg with
case)



1970's version (1.7kg)



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Modern versions

- These types of measurements are now directly built into the telescope assemblies of theodolites and you can see these on most construction sites. The angles are now also read electronically (compared to glass optical circles).
- Modern example (circa 2000)



Corner cube
reflector, Infrared
light source used



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Theory of EDM

- EDM is based on the idea that light (and radio waves) travel at a finite velocity and by measuring how long a signal takes travel back and forth between two points and knowing the speed of light, the distance can be measured.
- However, very few instruments actually make a time-of-flight measurement. Most instruments use a phase measurement (actually as series of such measurements). We will see shortly why.
- Start with time-of-flight because concept it is simple then move to phase (GPS actually uses both measurement types with some interesting twists).

Time of flight measurement

- In time of flight measurement, a pulse is transmitted and the time to the return is measured.
- Measurement can be made a number of ways:
 - Leading edge detection (signal level passes a threshold)
 - Centroid measurement (assume pulse shape of return)
 - “Matched filter”: outgoing pulse correlated with return pulse
- Accuracy of measurement depends on duration of pulse

Time-of-flight measurement

- If a “box-car” is transmitted (i.e., a rectangular pulse), correlation with another box car, will generate a triangular correlation function.
- The width of this function is twice the pulse length. A narrower pulse; the more precise the measurement.
- However a perfect box-car is impossible to generate because of the instantaneous rise time.
- Nature of pulse is accessed by Fourier transform of time-domain signal (i.e., its frequency content is determined).

Pulse characteristics

- The Fourier transform of a box car of height C and duration T seconds is:

$$\int_{-T/2}^{T/2} C e^{-i2\pi ft} dt = 4CT \frac{\sin(4\pi Tf)}{4\pi Tf}$$

- The function on the left is called the sinc function
- Notice that the width of the sinc function is $1/T$ (between zeros) and that its amplitude decays as $1/f$
- The equivalent width of a “pulse” is thought of as $1/(\text{frequency range})$ [called bandwidth]

Pulse characteristics

- Very narrow pulses, need a large frequency bandwidth and broad pulses require a small bandwidth (consider internet data transfer rates)
- In real systems bandwidth is limited by losses in the system that attenuate signals away from the center of the transmission frequencies (e.g., antennas only work around a certain frequency band).
- One of the advantages of optical frequencies is that since the frequency is so high (3×10^6 GHz compared to GPS at ~ 1 GHz (10^9 Hz))

EM Propagation

- Theory of propagation of EM (and interaction with antennas) is set by Maxwell's equations. We will not cover this area except to note that the solution to Maxwell's equations for a signal propagating in uniform, isotropic medium can be written as:

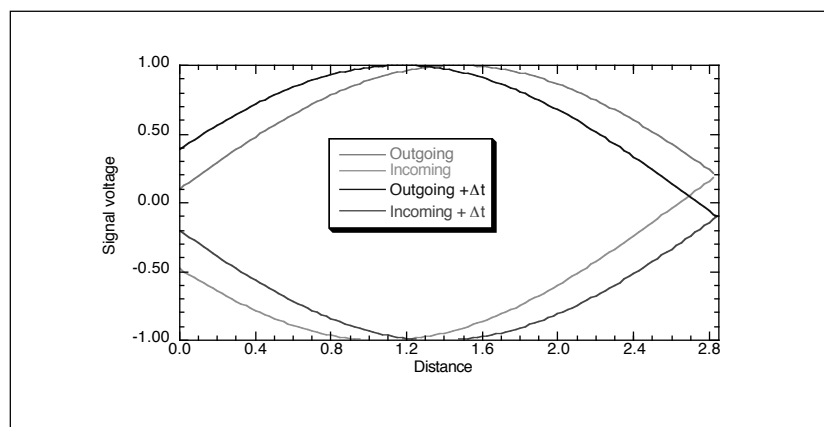
$$\vec{E}(\mathbf{x}, t) = \vec{E}_0 e^{-i2\pi(ft - \mathbf{k} \cdot \mathbf{x})}$$

- Where E is the electric field, t is time, \mathbf{x} is a position vector and \mathbf{k} is the wave vector (vector in direction of propagation divided by wavelength $\lambda = v/f$)

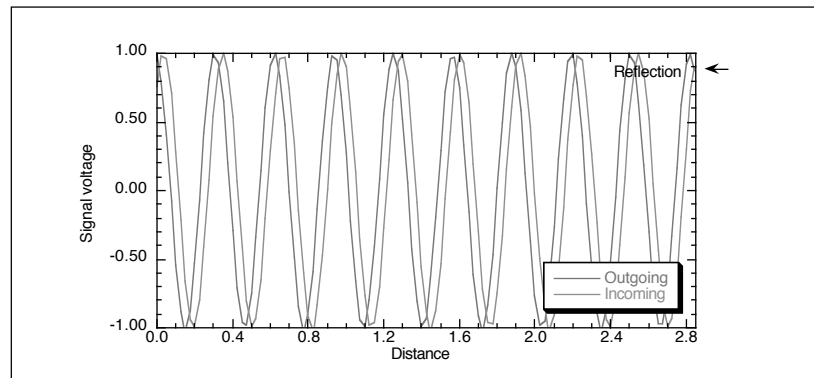
EM Receiver

- All an EM receiver does is sample the E field at a location (from measuring the current in a an antenna induced by the traveling E field) and convert it into a voltage that can be manipulated (e.g., AM and FM radio).
- If \mathbf{x} is fixed, then the $\mathbf{k} \cdot \mathbf{x}$ term is the phase of the observed signal. (The $2\pi f t$ term is removed by de-modulation i.e., multiplying by a signal of the same frequency).

Difference measurement (stays constant with time and depends on distance)



Higher frequency. Phase difference still says something about distance but how to know number of cycles?



Phase measurement of distance

- Phase difference between outgoing and incoming reflected tells something about distance
- If distance is less than 1 wavelength then unique answer
- But if more than 1 wavelength, then we need to number of integer cycles (return later to this for GPS).
- For surveying instruments that make this type of measurement, make phase difference measurements at multiple frequencies. (Often done with modulation on optical carrier signal).

Resolving ambiguities

- The range accuracy will be low for long-wavelength modulation: Rule of thumb: Phase can be measured to about 1% of wavelength
- For EDM: Use multiple wavelengths each shorter using longer wavelength to resolve integer cycles (example next slide)
- Using this method EDM can measure 10's of km with millimeter precision

Ambiguity example

- A typical example would be: Measure distances to 10 km using wavelengths of 20 km, 1 km, 200 m, 10 m, 0.5 m
- True distance 11 785.351 m

| Wavelength | Cycles | Resolved | Distance |
|------------|--------|----------|-----------|
| 20 km | 0.59 | 0.59 | 11800 |
| 1 km | 0.79 | 11.79 | 11790 |
| 200 m | 0.93 | 58.93 | 11786 |
| 10 m | 0.54 | 1178.54 | 11785.4 |
| 0.5m | 0.70 | 23570.70 | 11785.350 |

Frequency shifting

- In many EDM systems, the modulation frequency can be changed by small increments and this allows distances to be measured by setting the frequency to null the phase difference between the outgoing and incoming signal.
- We set the frequency such that $x/\lambda_1 = N$ (an unknown integer)
- If the frequency is slowly changed then the phase difference will be non-zero, but will return to zero again at some slightly different frequency so that $x/\lambda_2 = N+1$. How do we use this?

Frequency shifting

- If we are certain that λ_1 and λ_2 represent exactly one cycle difference over the distance x (requires fine tuning of frequency selector) then the two equations can be solved for x :

$$x / \lambda_1 = N$$

$$x / \lambda_2 = N + 1 \quad \text{Subtracting the two eqns}$$

$$x \frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} = 1 \quad \text{and} \quad x = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

- Many EDMs work this way but notice the sensitivity to the difference in wavelength

Application areas

- Commercial EDM equipment used by surveyors and engineers use the frequency changing systems (on a modulated signals and not the carrier frequency).
- Pulsed systems are used by radar and lidar (light detection and ranging)
- Satellite laser ranging (SLR) uses a pulsed system and is capable of getting return signals from the moon (Apollo experiment that still operates) and from earth orbiting satellites (LAGEOS and many others including some GPS satellites and all Russian GLONASS satellites)

Issues with EDM

- We have looked at a number of methods for measuring distances electronically. All have advantages and disadvantages:
 - Pulse systems: Pulse duration sets accuracy and to improve accuracy, average over a number of pulses. But transmitter not running most of the time (have to wait until pulse returns, low “duty-cycle”)
 - Phase measurement systems: require changing frequencies. OK for passive reflector (mirror) but limits the number of users if active return system (usual for radio systems)

Summary of Seminar

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