Review

• So far we have looked at measuring coordinates with conventional methods and using gravity field

• Today lecture:
  – Examine definitions of coordinates
  – Relationships between geometric coordinates
  – Time systems
  – Start looking at satellite orbits
Coordinate types

• Potential field based coordinates:
  – Astronomical latitude and longitude
  – Orthometric heights (heights measured about an equipotential surface, nominally mean-sea-level (MSL))

• Geometric coordinate systems
  – Cartesian XYZ
  – Geodetic latitude, longitude and height

Astronomical coordinates

• Astronomical coordinates give the direction of the normal to the equipotential surface

• Measurements:
  – Latitude: Elevation angle to North Pole (center of star rotation field)
  – Longitude: Time difference between event at Greenwich and locally
Astronomical Latitude

- Normal to equipotential defined by local gravity vector
- Direction to North pole defined by position of rotation axis. However rotation axis moves with respect to crust of Earth!
- Motion monitored by International Earth Rotation Service IERS [http://www.iers.org/](http://www.iers.org/)

\[ \phi_a = Z_d - \delta \]

Zenith distance = 90-elevation

\[ \delta \]

declination

To Celestial body

Rotation Axis
Astronomical Latitude

• By measuring the zenith distance when star is at minimum, yields latitude

• Problems:
  – Rotation axis moves in space, precession nutation. Given by International Astronomical Union (IAU) precession nutation theory
  – Rotation moves relative to crust

Rotation axis movement

• Precession Nutation computed from Fourier Series of motions
• Largest term 9″ with 18.6 year period
• Over 900 terms in series currently (see http://geoweb.mit.edu/~tah/mhb2000/JB000165_online.pdf)
• Declinations of stars given in catalogs
• Some almanacs give positions of “date” meaning precession accounted for
Rotation axis movement

- Movement with respect crust called “polar motion”. Largest terms are Chandler wobble (natural resonance period of ellipsoidal body) and annual term due to weather
- Non-predictable: Must be measured and monitored

Evolution (IERS C01)
Evolution of uncertainty

![Graph showing the evolution of uncertainty over time]

Recent Uncertainties (IERS C01)

![Graph showing recent uncertainties over time]
Astronomical Longitude

- Based on time difference between event in Greenwich and local occurrence
- Greenwich sidereal time (GST) gives time relative to fixed stars

\[
\begin{align*}
\text{GST} &= 1.0027379093UT1 + \frac{\dot{\vartheta}_0}{\text{GMST}} + \Delta \psi \cos \varepsilon \\
\dot{\vartheta}_0 &= 24110.54841 + 8640184.812866 T + 0.093104 T^2 - 6.2 \times 10^{-6} T^3
\end{align*}
\]
Universal Time

- UT1: Time given by rotation of Earth. Noon is “mean” sun crossing meridian at Greenwich
- UTC: UT Coordinated. Atomic time but with leap seconds to keep aligned with UT1
- UT1-UTC must be measured

Length of day (LOD)

LOD = Difference of day from 86400. seconds
Recent LOD

LOD compared to Atmospheric Angular Momentum
LOD to UT1

- Integral of LOD is UT1 (or visa-versa)
- If average LOD is 2 ms, then 1 second difference between UT1 and atomic time develops in 500 days
- Leap second added to UTC at those times.

UT1-UTC

- Jumps are leap seconds, longest gap 1999-2006. Historically had occurred at 12-18 month intervals
- Prior to 1970, UTC rate was changed to match UT1
Transformation from Inertial Space to Terrestrial Frame

• To account for the variations in Earth rotation parameters, as standard matrix rotation is made

\[
\begin{array}{cccc}
\chi_i & \rightarrow & P & N & S & W & \chi_t \\
\text{Inertial} & \text{Precession} & \text{Nutation} & \text{Spin} & \text{Polar Motion} & \text{Terrestrial}
\end{array}
\]

Geodetic coordinates

• Easiest global system is Cartesian XYZ but not common outside scientific use
• Conversion to geodetic Lat, Long and Height

\[
\begin{align*}
X &= (N + h)\cos \phi \cos \lambda \\
Y &= (N + h)\cos \phi \sin \lambda \\
Z &= \left(\frac{b^2}{a^2} N + h\right) \sin \phi \\
N &= \frac{a^2}{\sqrt{a^2 \cos^2 \phi + b^2 \sin^2 \phi}}
\end{align*}
\]
Geodetic coordinates

• WGS84 Ellipsoid:
  – $a=6378137$ m, $b=6356752.314$ m
  – $f=1/298.2572221 = [a-b]/a$
• The inverse problem is usually solved iteratively, checking the convergence of the height with each iteration.
• (See Chapters 3 & 10, Hofmann-Wellenhof)

Heights

• Conventionally heights are measured above an equipotential surface corresponding approximately to mean sea level (MSL) called the geoid
• Ellipsoidal heights (from GPS XYZ) are measured above the ellipsoid
• The difference is called the geoid height
Geoid Heights

- National geodetic survey maintains a web site that allows geoid heights to be computed (based on US grid)
  - http://www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/geoid99_prompt1.prl
- New Boston geoid height is -27.688 m

NGS Geoid 99 http://www.ngs.noaa.gov/GEOID/GEOID99/
Spherical Trigonometry

• Computations on a sphere are done with spherical trigonometry. Only two rules are really needed: Sine and cosine rules.

• Lots of web pages on this topic (plus software)
  • http://mathworld.wolfram.com/SphericalTrigonometry.html is a good explanatory site
Basic Formulas

A B C are angles
a b c are sides
(all quantities are angles)

Sine Rule
\[ \frac{\sin a}{\sin b} = \frac{\sin b}{\sin A} = \frac{\sin c}{\sin C} \]

Cosine Rule sides
\[ \cos a = \cos b \cos c + \sin b \sin c \cos A \]
\[ \cos b = \cos c \cos a + \sin c \sin a \cos B \]
\[ \cos c = \cos a \cos b + \sin a \sin b \cos C \]

Cosine Rule angles
\[ \cos A = -\cos B \cos C + \sin B \sin C \cos a \]
\[ \cos B = -\cos A \cos C + \sin A \sin C \cos b \]
\[ \cos C = -\cos A \cos B + \sin A \sin B \cos c \]

Basic applications

• If b and c are co-latitudes, A is longitude difference, a is arc length between points (multiply angle in radians by radius to get distance), B and C are azimuths (bearings)
• If b is co-latitude and c is co-latitude of vector to satellite, then a is zenith distance (90-elevation of satellite) and B is azimuth to satellite
• (Colatitudes and longitudes computed from \( \Delta XYZ \) by simple trigonometry)
Summary of Coordinates

- While strictly these days we could realize coordinates by center of mass and moments of inertia, systems are realized by alignment with previous systems.
- Both center of mass (1-2cm) and moments of inertia (10 m) change relative to figure.
- Center of mass is used based on satellite systems.
- When comparing to previous systems be cautious of potential field, frame origin and orientation, and ellipsoid being used.