Problem Set 3

1) Compute the amplitude and phase spectra for the examples given in
   the class notes [square wave, sawtooth, "rectified"
   sine wave, delta function]. Express the answers as formulae and
give asymptotic results for large n in terms of a single power of
n (e.g. $n^{-2}$).

2) Consider the "cosine bell" function

$$f(x) = \begin{cases} 
1 + \cos \frac{\pi x}{\delta} & -\frac{\delta}{2} \leq x \leq \frac{\delta}{2} \\
0 & \text{otherwise}
\end{cases}$$

Assume periodicity with "unit cell" of length L.

a) Derive the Fourier coefficients $a_n$ and $b_n$.

b) Write formulae for $a_n$ and $b_n$ for the special case $\delta = L$.

c) For the special case $\delta = L/2$, derive a simple form for the
   coefficients. Compare this form with the form for the
   similarly shaped "rectified" sine wave. How are the spectra
   different? Why?

d) Compute and plot the amplitude and phase spectra for $\delta = L/10$
   and $n \leq 50$. Comment on the shape of the spectrum in terms of
   that for a delta function.

e) Using your results from d), plot the function expanded over
   the ranges:

   \begin{align*}
   0 & \leq n \leq 20 \\
   0 & \leq n \leq 10 \\
   11 & \leq n \leq 20
\end{align*}

Comment on the physical reality of the features in each plot.