Review of last Class

• Topics to be covered will be those later in the course
• General areas are:
  – Vectors and matrices
  – Solving linear equations
  – Vector Spaces
  – Eigenvectors and values
  – Rotation matrices
Today’s class

- Analysis of Sextant measurements
- Homework was broken into a number of small steps:
  - Determining the maximum observed angle to the sun and time this maximum occurred
  - Obtaining the mean index error
  - Computing maximum elevation to the sun
  - Computing the atmospheric bending correction
  - Computing the latitude
  - Computing the longitude

Simpler parts of calculation

- **Mean of index error**: Simply the sum of the values divided by the number of values
- Also we can compute a standard deviation about the mean (also called a root-mean-square (RMS) scatter). This gives an indication of how well we can make measurements with the sextant. The standard deviation of our measurements was 1.5’
- We use this today and in later lectures we will show how to use this to allow us to estimate the uncertainty of our final latitude and longitude determination.
Atmospheric refraction

• We can use the simple formula given in class or we can look up the values in the Nautical Almanac.
• The formula result is slightly greater than 1’ since \( \tan(\varepsilon) \sim 1 \)
• Using the almanac we can explore how much this value will vary due to atmospheric conditions.
• (For latitude determination, atmospheric refraction becomes a bigger problem the closer we get to the pole where the meridian crossing elevation angle will be much smaller. It will also be a bigger problem in mid-winter than in mid-summer).

Geometry of measurement

• Spherical trigonometry that we can solve (we interpret on the meridian and so easy)
Spherical Trigonometry

• Based on the figure, we can write the solution for the zenith distance to the sun:

\[
\cos Zd = \cos \theta \cos(90 - \delta) + \sin \theta \sin(90 - \delta) \cos(\Delta GHA)
\]

• If we assume we know our latitude in longitude then we can compute the expected variations in the zenith distance to the Sun
• In addition, since we measured 2*(elevation to sun+refraction)+ index error, we can include this in what is called a “forward model”

Results of forward model

• GPS latitude 42.36; longitude -71.0890
• Declination of Sun at MIT meridian crossing -8.07 deg
• Interpolating the Almanac GHA, UT meridian crossing 16.5087 hrs (-4 hrs to EST)
• The forward model can be computed and compared to measurements.
• (Since index error close and opposite sign to refraction we can neglect at the moment).
Comparison

- Agreement looks good but when totals are displaced the results can be be deceptive in that the details cannot be seen.
- Normal to look at the difference between the observations and the model.
- On the quadratic fit residuals we show “error bars” based on the index measurements. These are computed from \( \sqrt{\text{Sum(residuals}^2)/(\text{number}-1)} \). Also called Root-mean-square (RMS) scatter.
- In class we will vary the parameters of the model to see their effect on the fit to the data.
Some neglected effects

- Refraction and index error not included in forward model.
- Motion of Sun during measurements was accounted for during the run.
- Later we will use the forward model to obtain rigorous estimate of latitude and longitude.
Summary:

• Today we explored the latitude and longitude problem in more detail looking at the actual data collected with the sextant.

• Introduced the notion of a forward model for comparing with data and varying the parameters of the model to better match the observations.

• Differences between observations and models can be quantified with an estimated standard deviation or RMS scatter.

• These issues are returned to when we address statistics and estimation.