Off-Nadir Atmospheric Delay Corrections Addendum

The atmospheric delay calibration ATBD discussed the effects of off-nadir pointing on the atmospheric delay correction and concluded that for pointing within 15° of nadir, a simple 1/sin(elevation angle) formulation would be provide a sub-millimeter accuracy corrections. Since it now seems likely that ICESat might point as far as 35° off-nadir, we have revaluated this approximation for large off nadir angles (up to 35°). The geometry for relating off-nadir angle, \( q \), to zenith angle, \( z = 90 \degree - e \), where \( e \) is the elevation angle, is shown in Figure 1.

![Figure 1: Geometry relating nadir angle to elevation angle.](image)

The comparison here we used the Niell [1996] hydrostatic mapping function that relates the atmospheric delay in the zenith direction to the delay at a specified (in-vacuum) elevation angle. The form of the mapping function is a continued fraction in \( \sin(e) \). With the coefficients appropriate for polar regions the Niell mapping function takes the form:
\[ DL = m(e)D_{L_z} \]

\[ m(e) = \frac{1/(1 + a/(1 + b/c))}{\sin(e) + a/(\sin(e) + b/(\sin(e) + c))} \]  

(1)

where \( DL \) is the atmospheric delay at elevation angle, \( D_{L_z} \) is the delay in the zenith direction. For Polar Regions, under average conditions, \( a=1.2046 \times 10^{-3}, b=2.90249 \times 10^{-3}, \) and \( c=64.258 \times 10^{-3} \).

Figure 2 shows the values of \( DL \), for \( D_{L_z}=2.3 \) m as a function of off-nadir angle for \( m(\theta) \) given in equation (1) and \( m(\theta) \) given simply by \( 1/\sin(\theta) \). At this scale, the differences are difficult to see. In Figure 3, we show the difference in units of mm. At the largest off-nadir angle, the difference is <2.5 mm and well within the atmospheric delay model error budget.

Figure 2: Atmospheric delay as function of off-nadir angle under nominal conditions given in the text. Units are meters.
Bending effects

The effects of bending were approximately evaluated in the ATBD for off-nadir angles up to 15°. We have more carefully considered these effects here because the off-nadir angles could be as large as 35°. To evaluate the effect we ray-traced through a standard, spherically symmetric, atmospheric model keeping careful track of the bending angles and the deviation between the vacuum and refracted paths. The ray tracing was performed from the ground to the satellite (at 600 km altitude) since this approach tends to more numerically stable than ray tracing from vacuum into the Earth’s atmosphere.

The ray tracing started at a series of elevations ranging between 90 and 50 degrees. From the ray-trace, the nadir angle at the satellite and the angle subtended by the arc between initial starting point and the position of ray when it reached 600 km were computed. In addition, we also integrated the atmospheric delay and the bending angle as checks on the ray-trace. The differences between the atmospheric delays computed from the ray-trace and those given by the simply cosecant law were indistinguishable from those shown in Figure 3 above. The bending angle matched the values given be the Astronomical Almanac [1999] formula in the ATBD to within one milli-degree.

Figure 4 show the arc distance to the footprint from the sub-satellite point computed from the ray tracing and from simple in vacuum geometry. Figure 5 shows the difference. In the worst case, the difference in foot print location is less than 5 meters and we conclude that atmospheric bending effects on foot print location can be ignored even for the largest
off nadir pointing angles. Intuitively these results make sense when it is realized that most of the bending occurs in the lowest 10km of the atmosphere, and for a bending angle of 0.014 degrees from 10 km altitude, the footprint displacement would be 3.2 m for a ray at 50° deg elevation angle.

Figure 4 also shows that for a 94° inclination orbit (sub satellite point ~440 km from the pole, that a nadir angle of ~35° will be needed to range to the pole.

Conclusions

Based on these calculations, we recommend that a simple $1/\sin(\theta)$ where $\theta$ is given by $\cos^{-1}(\sin(\theta)R_g/R_s)$ and $R_g$ is the radius at the footprint and $R_s$ is the radius to ICESat. To <5 meters, the bending of the ray by the atmosphere can be neglected in geo-locating the footprint.

Figure 4: Distance between the footprint location and the sub-satellite point as a function of nadir angle computed in vacuum and by ray tracing through a standard atmospheric delay model.
Figure 5: Difference between the two curves in Figure 4 shown in meters.

References