GLAS Team Member Quarterly Report

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General

Tom Herring has been analyzing the GPS data collected on the helicopter doing laser altimetry imaging after the Hector Mine earthquake. This data set poses a number of problems in that there are trajectories for which there is no stationary GPS data available (i.e. all of the GPS data is from the flight portion of the mission. The data also has a large number of gaps in it which seem to result from recording problems rather than tracking problems in the Ashtech receivers used.

Katy Quinn continues to work on her thesis. The sections dealing with the ICEsat atmospheric delay calibration are largely finished. The comparison of the algorithm results with surface barometers and total atmospheric delay inferred from GPS data indicates that the algorithm should have no difficulty in meeting the 20 mm error budget for GLAS.

In this quarterly, we focus on the work being done by graduate student An Nguyen on the applications of block krigging to measuring areal average height changes with GLAS.

Block krigging and power spectral densities of generated GLAS data

Introduction

As a continuation of the previous study on ordinary krigging as a method of estimating elevation changes and errors, I proceed to estimate these parameters using block krigging. This is similar to a study done by K. Quinn on the MOLA, but aiming at trying to estimate changes and errors at sub-meter level. In addition I also introduced time variation into the “data” at different frequencies and attempted to recover these input signals from the power spectral density of the krigged and averaged results.

Block Krigging

In dealing with a large set of data, block krigging has the advantage over ordinary krigging. Similar to the steps in ordinary krigging, we obtain the weights \( w_j \) in order to calculate the estimate \( \hat{v} \) of the blocks (Olea, 1999). Given the set of data \( \{v_1, v_2, ..., v_N\} \) inside a search neighborhood, the covariance functions \( C_v \) and \( C_V \) between the \( k_{th} \) block of area \( A \) and its surrounding data points \( v_i, i \leq N \) and blocks \( V_j \) are respectively defined as,

\[
C_v_{ik} = \frac{1}{A_k A_i} \int C_{il} dA_l
\]

\[
C_V_{jk} = \frac{1}{A_k A_j} \int \int C_{mn} dA_m dA_n
\]
where $C_{ij}$ is the covariance function between two points $v_i$ and $v_j$. The equations for the block estimates thus will follow those for point kriging,

$$
\begin{bmatrix}
C_{11} & \cdots & C_{1N} & 1 \\
\vdots & \ddots & \vdots & \vdots \\
C_{N1} & \cdots & C_{NN} & 1 \\
1 & \cdots & 1 & 0
\end{bmatrix}
\begin{bmatrix}
w_1 \\
\vdots \\
w_N \\
\mu
\end{bmatrix}
= 
\begin{bmatrix}
Cv_{1k} \\
\vdots \\
Cv_{Nk} \\
1
\end{bmatrix}
$$

or

$$B \cdot L = D$$

where $\mu$ is the Lagrange parameter to ensure the unbiased constraint, $\sum_{i=1}^{N} w_i = 0$. Thus,

$$L = B^{-1} \cdot D$$

$$\Rightarrow \hat{V}_k = \sum_{i=1}^{N} w_i \cdot v_i$$

$$E[\hat{V}_k - V_k] = 0$$

$$\sigma_{\hat{V}_k}^2 = E[(\hat{V}_k - V_k)^2] = CV_{kk} - B^{-1} \cdot D$$

**Power Spectral Density Estimates of Temporal Variations**

Power spectral density (PSD) curves for a time sequence are obtained using two methods. In a classical approach using the Blackman-Tukey indirect method (Kay & Marple, 1981), the auto-correlation function $\Phi_{xx}(m)$ of the time-dependent signal is pre-multiplied with a window $W[n]$, $Hanning_2$ in this case (Harris, 1978), then Fourier transformed to obtain its PSD $\hat{\Phi}(f)$,

$$\hat{\Phi}_{xx}(m) = \frac{1}{N} \sum_{n=0}^{N-m-1} x_{n+m} x_n, x \in \mathbb{R}$$

$$W[n] = \cos^2 \left[ \frac{n \pi}{K} \right], n = -\frac{K}{2}, \ldots, 1, 0, 1, \ldots, \frac{K}{2}$$

$$\hat{\Phi}(f) = \Delta t \sum_{n=-M}^{M} \Phi_{xx}(m) \exp(-j2\pi fm\Delta t)$$

where $\Delta t$ is the sampled time and $-\frac{1}{2\Delta t} \leq f \leq \frac{1}{2\Delta t}$.

In the modern parametric approach, one assumes a model for the time signal and try to estimate the parameters of the model (Kay & Marple, 1981). One can assume an input signal of white noise $w[n]$ with distribution $N[0,1]$ passing through a linear filter $h[n]$ to produce the output time sequence $v[n]$. The task is then to estimate the coefficients for $h[n]$ in its z-domain. In an
auto-regressive model, $H(z)$ is modeled as an all-pole filter $\frac{1}{A(z)}$. The mathematical formulation is as follows,

$$h[n] \leftrightarrow H(z) = \frac{1}{A(z)}$$

$$A(z) = \sum_{m=0}^{p} a[m] z^{-m}$$

$$z = \exp(j2\pi f\Delta t)$$

$$v[n] = h[n] \cdot w[n] = w[n] - \sum_{k=1}^{p} a[k] x[n-k]$$

$$PSD_v = \hat{\Phi}(f) = \frac{\Delta t}{|A(f)|^2}$$

One solves for the coefficients $a[k]$ by relating to the auto-correlation function $\Phi_{vv}[m]$ of $v[n]$ to obtain a system of equations known as the Yule-Walker equations,

$$\Phi \cdot A = D$$

where

$$\Phi = \begin{bmatrix} \Phi_{vv}[0] & \Phi_{vv}[-1] & \ldots & \Phi_{vv}[-p] \\
\Phi_{vv}[1] & \Phi_{vv}[0] & \ldots & \Phi_{vv}[-(p-1)] \\
\vdots & \vdots & \ddots & \vdots \\
\Phi_{vv}[p] & \Phi_{vv}[p-1] & \ldots & \Phi_{vv}[0] \end{bmatrix}$$

$$A = [a[0], a[1], \ldots, a[p]]^T$$

$$D = [1, 0, \ldots, 0]^T$$

Thus one can solve for the coefficients in $A$ by inverting $\Phi$,

$$A = \Phi^{-1} \cdot D$$

and obtain $A(z)$ and the PSD of $v[n]$.

**Results**

Projected topography from $GTOPO30 DEM$ based on the approximate 183-day repeat cycle at $1$ Hz resolution was used as “data”. At this testing phase, krigged results of single approximately 100 km x 100 km blocks were generated for analysis of errors and time variations. Initial time dependence was introduced with a combination of a semi-annual signal of amplitude 5 meters, a high frequency random fluctuation of amplitude uniformly distributed between zero and one and period 0.1 year, and a long term change of amplitude 2-meter over 20 years (Fig.1),

$$v = v_o + 5\sin\left(\frac{2\pi t}{0.5}\right) + rand[0, 1] \cdot \sin\left(\frac{2\pi t}{0.1}\right) + 2\sin\left(\frac{2\pi t}{40}\right)$$

where $rand[0,1]$ represents uniformly distributed random numbers between zero and one.

Because we are looking for long term changes in the volume of the ice sheets, it is reasonable to view the 25-day sub-cycle data set as a snap shot and represent its krigged/averaged result...
as one point on the time axis (Fig.2). I have generated two years worth of time dependent data to obtain approximately 30 data points (Fig.2-3). Comparison of krigged versus straight spatial averaging results shows consistently higher values of about 0.4 m in the krigged maps with a temporal residual of mean 0.46m and variance 0.0014m². Such low variance implies that this offset between krigged and averaged results is systematic. The banded pattern in the spatial residual map between krigged and averaged results (mean 0.46m, variance 2.7m²) reflects different ratios of data points between the blocks, e.g., blocks at cross-over latitudes tend to have more data (Fig 4c-d). However I believe this is an artificial effect of the generated ground-track map (not exactly matching the currently available 183-day repeat tracks). This banded pattern is more visible in data sets with high latitudinal variations. Comparison between the residual map (Fig. 4c) and data variance within the sub-blocks (not shown here) shows that residual is high in the upper corner of the block where data variance is in the range of 400-600 m² (compared to 100-200 m² everywhere else in the block).

Due to the treatment of the 25-day data set as a snap shot, we have essentially smoothed out the high frequency component of the time signal (Fig. 3). As a result, its PSD does not show a peak corresponding to the input frequency of 10 Hz. The expected peak at input frequency of 0.025 Hz is missing possibly due to two reasons: the unstable mean/krigged elevation at the beginning of the record due to lack of spatial coverage, and the lack of data points in the time sequence. In addition to a peak at input frequency of 2Hz, there is also one at frequency 0.5Hz that is left unaccounted for. One possible explanation for this extra peak is the artificially slow damping of the topography with time as it approaches its stable mean. Such peaks could possibly be eliminated by discarding the first one half year of data because of the lack of spatial coverage.

**In progress / Future works**

Work in progress currently includes obtaining krigged/averaged maps over rough terrains and studying their PSD’s. It will also be useful to look at the PSD of the difference between krigged and averaged results, and PSD of differences between temporal krigged values (Herring, 2001). I am also looking into results of snow-fall rates, wind directions, and terrain distribution in the literature in order to implement a more realistic spatial-temporal variations into the topography data over Antarctica. Results of the krigged variance are also being analyzed. However these variance maps only reflect how good the spatial coverage is and not how well the krigged results are compared to actual “data”. Another related task is to ‘roughen up’ the data to a resolution comparable to that of the footprints (~70m x 170m). Chou (1995) had done some works previously on introducing spatial-temporal variations using cross-over data.

**Reference**

Figure 1. Temporal variation components which include a short term fluctuation of period $T=0.1\,\text{yr}$, amplitude $A=[0,1]\,\text{m}$, a semi-annual term of $T=0.5\,\text{yr}$, $A=5\,\text{m}$, and a long term trend of $T=40\,\text{yr}$, $A=2\,\text{m}$
Figure 2. “Snap shots” at 25-day sub-cycle interval showing GLAS generated ground-tracks for block $[\text{lon, lat}]=[80, 85, -81, 80]$. At each time-step the number of data points used for krigging is cumulative. Thus at the last time-step (step 30 -- $t=2\text{yr}$) all data points is included.
Figure 3. (a) Time sequence of krigged and averaged results for block /80,85,81,80/ with interpolated values at sampled time $\Delta t=0.01\text{yr}$. (b) PSD of the time sequence in (a) showing peak at input frequency $f=2\text{Hz}$, missing peaks at input frequencies $f=0.025\text{Hz}$ & $f=10\text{Hz}$, and peaks unaccounted for at other frequencies. (c) Temporal residual curve between krigged and averaged results. The very low variance in the residual is indicative of a systematic difference between the two results.
Figure 4. Krigged (a) and Averaged (b) results for block $[80, 85, -81, -80]$ averaged over 30 time-steps with their spatial residual shown in (c). The banded pattern in the residual map is probably related to the banded pattern in the number of data points at each sub-block (d). High residual occurs where data variance within the corresponding sub-blocks are high. The latitudinal dependence of number of data points is believed to be an artificial effect of location of cross-overs (see Fig. 2).