GLAS Team Member Quarterly Report

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Period: 01/01/2003 to 03/31/2003

General:

During this quarter, An Nguyen has continued her analysis of ERS data using methods that will be applied to the ICESat data once the 183-day repeat cycle has started. It is expected that she will be able to apply these methods to about 1-year of ICESat data before graduating. The algorithms that she develops will then continue to be applied to the ICESat data most likely while An completes a postdoctoral fellowship.

We have also re-visited the issue of the atmospheric delay calibration for large (up to 35 degrees) off-nadir pointing angles. Off-nadir effects were addressed in our original ATBD but angles only as large as 10 degrees were considered (at the time this was the specification). The new and more complete analysis verifies our original conclusions even at the large nadir angles that a simple scaling of the zenith atmospheric delay by 1/sin(elevation angle) is adequate approach to computing the off-nadir atmospheric delay. Here the elevation angle is the elevation angle, at the ground to the satellite, computed using in-vacuum geometry. With this approximation, the footprint location should not be in error by more than 4 meters and the computed atmospheric delay should not be in error by more than 2.5 mm.

We are now examining the early ICESat data. There studies concentrate in two areas:
(a) Validation of the atmospheric delays being computed by the ISIP for the GLA06 data product;
(b) Strength of reflected pulses in an effort to assess the probability of a near-normal specula reflection damaging the GLAS receiver. We are using three different approaches for this study. The initial results will be presented at the ICESat CAL/VAL meeting on April 22-23. The approaches are:
(b.1) Examine the statistics of the integrated power under the fitted waveform and then renormalize these results for maximum receiver gain and a single 2.3 ns width pulse. The basis of this algorithm is that the non-linear receiver behavior of the receivers conserves the total power received.
(b.2) Examine the statistics of the peak voltage detected by the receiver. This estimate is likely to be conservative because of the clipping by the detectors.
(b.3) Based on ICESat elevation data over the oceans and ice sheets combined with significant wave height estimates from TOPEX, examine in detail areas that are smooth (smooth shot-to-shot behavior) to assess the possible surface slopes that might exist and compare the distribution of these values with the 0.0068 radians off-nadir pointing of ICESat.

The remainder of this report in divided into a section on atmospheric delays with off-nadir pointing and a section on the ERS analysis.
Off-Nadir Atmospheric Delay Corrections Addendum

The atmospheric delay calibration ATBD discussed the effects of off-nadir pointing on the atmospheric delay correction and concluded that for pointing within 15° of nadir, a simple $1/\sin(\text{elevation angle})$ formulation would be provide a sub-millimeter accuracy corrections. Since it now seems likely that ICESat might point as far as 35° off-nadir, we have revaluated this approximation for large off nadir angles (up to 35°). The geometry for relating off-nadir angle, $\theta$, to zenith angle, $z=90-\text{elevation angle}$, is shown in Figure 1.

![Figure 1: Geometry relating nadir angle to elevation angle. R is the radius of the Earth (~6378 km); h is the altitude of the satellite (~600km); $\theta$ is the nadir angle; and z is the zenith angle, which equals 90-$\theta$ where e is the elevation angle. For $\theta=35^\circ$, $z=38.868^\circ$ and $e=51.132^\circ$.](image)

The comparison here we used the Niell [1996] hydrostatic mapping function that relates the atmospheric delay in the zenith direction to the delay at a specified (in-vacuum) elevation angle. The form of the mapping function is a continued fraction in $\sin(e)$. With the coefficients appropriate for polar regions the Niell mapping function takes the form:
\[
\begin{align*}
\square L &= m(\square L_z) \\
m(e) &= \frac{1/(1 + a/(1 + b/1 + c)))}{\sin(e) + a/(\sin(e) + b/(\sin(e) + c)))}
\end{align*}
\]

where \(\square L\) is the atmospheric delay at elevation angle, \(\square L_z\) is the delay in the zenith direction. For Polar Regions, under average conditions, \(a=1.2046 \times 10^{-3}\), \(b=2.90249 \times 10^{-3}\), and \(c=64.258 \times 10^{-3}\).

Figure 2, shows the values of \(\square L\), for \(\square L_z=2.3\text{m}\) as a function of off-nadir angle for \(m(\square)\) given in equation (1) and \(m(\square)\) given simply by \(1/\sin(\square)\). At this scale, the differences are difficult to see. In Figure 3, we show the difference in units of mm. At the largest off-nadir angle, the difference is <2.5 mm and well within the atmospheric delay model error budget.
Bending effects

The effects of bending were approximately evaluated in the ATBD for off-nadir angles up to 150°. We have more carefully considered these effects here because the off-nadir angles could be as large as 35°. To evaluate the effect we ray-traced through a standard, spherically symmetric, atmospheric model keeping careful track of the bending angles and the deviation between the vacuum and refracted paths. The ray tracing was performed from the ground to the satellite (at 600 km altitude) since this approach tends to more numerically stable than ray tracing from vacuum into the Earth’s atmosphere. The ray tracing started at a series of elevations ranging between 90 and 50 degrees. From the ray-trace, the nadir angle at the satellite and the angle subtended by the arc between initial starting point and the position of ray when it reached 600 km were computed. In addition, we also integrated the atmospheric delay and the bending angle as checks on the ray-trace. The differences between the atmospheric delays computed from the ray-trace and those given by the simply cosecant law were indistinguishable from those shown in Figure 3 above. The bending angle matched the values given by the Astronomical Almanac [1999] formula in the ATBD to within one milli-degree.

Figure 4 show the arc distance to the footprint from the sub-satellite point computed from the ray tracing and from simple in vacuum geometry. Figure 5 shows the difference. In the worst case, the difference in foot print location is less than 5 meters and we conclude that atmospheric bending effects on foot print location can be ignored even for the largest off nadir pointing angles. Intuitively these results make sense when it is realized that most of the bending occurs in
the lowest 10km of the atmosphere, and for a bending angle of 0.014 degrees from 10 km altitude, the foot print displacement would be 3.2 m for a ray at 50° deg elevation angle.

Figure 4 also shows that for a 94° inclination orbit (sub satellite point ~440 km from the pole, that a nadir angle of ~35° will be needed to range to the pole.

**Conclusions**

Based on these calculations, we recommend that a simple $1/\sin(\theta)$ where $\theta$ is given by $\cos^{-1}(\sin(\theta)R_{g}/R_{s})$ and $R_{g}$ is the radius at the footprint and $R_{s}$ is the radius to ICESat. To <5 meters, the bending of the ray by the atmosphere can be neglected in geo-locating the footprint.

**Figure 4:** Distance between the foot print location and the sub-satellite point as a function of nadir angle computed in vacuum and by ray tracing through a standard atmospheric delay model.
Figure 5: Difference between the two curves in Figure 4 shown in meters.

References
An Application of Kriging Method to the Detection of Elevation Changes with Satellite Altimetry Data
Nguyen, A.T. and T.A. Herring

Abstract
We use kriging to investigate elevation changes in Antarctica using ERS-1/2 data from the years 1994 to 2001. Residuals between ERS radar altimetry and GTOPO30 DEM follow variograms of the form $\Gamma(h) = a^2 h^b$. Our analyses show a strong anisotropy in $\Gamma$ but weaker one in $\lambda$, with $\lambda$ in the range $[0.4,0.7]$ and $\Gamma [0.6,1.3]$. Temporal kriged blocks at approximately 100km x 100km with time steps of 35-day using 250-point nearest neighborhood show a dominant annual signal of amplitude range $[0.13,0.36]$ which could be partially explained for by firn compaction. Fitted trends in the time sequences reveal a stable eastern Antarctica, with a positive rate of change $dh/dt$ of approximately 47±40 cm/yr to 96±62 cm/yr across the head of the Amery Ice Shelf and large changes at the coast and in western Antarctica. RMS deviations from trends are small for the stable interior eastern Antarctica, with large RMS near the Amery Shelf, edge of the continent, and in western Antarctica.

Introduction
This study is a continuous work of the previous report where we investigated the use of kriging on ERS-1 residuals to study elevation changes in a small region in western Antarctica [Nguyen and Herring, 2002]. Since then we have successfully analyzed four regions covering approximately all of eastern Antarctica north of -82° and a small part in western Antarctica for all available ERS data beginning April 10, 1994.

Data
ERS-1 168-day and 35-day repeat cycles beginning April 1994 and ERS-2 35-day repeat cycles beginning June 1996 were used in this study (Table.1). Data corrections include removing waveforms with high noise, bad orbits, undefined slope correction, tidal effect as summarized in Table.2 (about 6.5% of the data). GTOPO30 DEM was used as a-priori map to obtain elevation residuals. An approximately 35-day time-step is used for time series analyses.

Results & Discussion

Anisotropic variograms
Variogram structure of the form $\Gamma(h) = a^2 h^b$ where $a^2$ is the residual variance, $h$ the distance in degrees, and $a$, $b$ the parameters which characterize the region of interest, was fitted to ERS residuals using least linear square inversion. With the removal of bad orbital corrections, residual variances for each of the four regions shown in Figure.1 are approximately constant with time. Bad orbital corrections are defined as those whose sum within the individual orbits exceed 0.5 meter. For all regions correlations are relatively high at small distances, with nearly no nugget effect.

Figure.2 shows three scenarios of $[\lambda,\Gamma]$. Case A (small $\lambda$, large $\Gamma$) implies high correlation at small $h$ with significant drop-off in correlation with distance (large correlation length). Case C (large $\lambda$, small $\Gamma$) implies rapid approach to uniformly distributed random processes with potential nugget effect at very small $h$. Case B represents the transition between cases A and C.
West Antarctica - Region C1500908272

Parameters [a] and [b] in this region consistently shows directional dependency with [h_{min},h_{max}] of [0.45,0.63] in directions [N90E,N0E], and [h_{min},h_{max}] of [0.55,0.7] in directions [N45E,N120E] respectively (Fig.1, Fig.3). Fig.2 suggests that elevation residuals in this region have higher correlation near h=0 in East-West than North-South directions (min vs. max [a]), and correlation length is higher in N120E than N45E (max vs. min [b]). This anisotropy suggests a possible correlation with the topography in the region (Fig.1), which exhibits an X-pattern of alternative highs and lows in directions approximately N45W and N45E respectively. However this would imply a systematic bias in the GTOPO DEMs.

East Antarctica

In East Antarctica, the Amery Ice Shelf region C0500908171 shows a dominant anisotropic pattern with [a] in the N90E direction. [b] varies less with directions in the two other regions C0100408171 and C1001408167 (Fig.1, Fig.3). Parameter [a] appears to be nearly isotropic except the sharp decreases in N30E and N30W which could be related to the along-track directions. As spatial resolution is higher along track, a low [a] value suggests either a) correlation length is short (case C in Fig.2), or b) the data variance is somehow smaller along track than in any other directions. In case a) we would expect the corresponding [b] to be high. However there is no observable correlation between [a] and [b] anisotropies. If case b) is likely, it suggests a systematic bias in the data collection process. There is no visible correlation between topography gradient and [a] and [b] in East Antarctica.

Kriging

As described in our previous report [Nguyen and Herring, 2002], elevation residuals between ERS and GTOPO30 DEM were binned into approximately 35-day time steps. We kriged and averaged at approximate resolution grid 0.5° by 0.5° using nearest 250-point neighborhood. Both kriged and average results contain dominant annual signals of amplitudes ranging from approximately 0.13 to 0.36 meter. Once the annual signal was removed, we fitted the results to straight lines to obtain the trends and RMS deviation from trends. Low RMS implies stable blocks in the interior of the ice sheet.

Figures 5 and 6 show the resultant trends and RMS deviations overlaying topographic maps of Antarctica. Trends are possible indications of long term elevation changes. The regions in eastern Antarctica seem stable with near zero trends and RMS, except at the head of the Amery Shelf which shows both positive trend of approximately 47±40 cm/yr to 96±62 cm/yr with corresponding RMS of 4m to 12m. Region C0100408171 shows evidence of positive elevation changes near the coast; however the signals are high with again large RMS. As expected RMS increases toward the edge of the ice sheet and in regions close to ice shelves. An increase of approximately 14±4 cm/yr to 39±4 cm/yr is consistent with those reported by Zwally et al. [2002]. However our results did not show the dominant decrease centered near [77,-105]. Alternate negative and positive trends in this region in west Antarctica are likely indications of the instability. The difference between kriged and average results are currently being analyzed.

During analyses of the time sequence of kriged blocks, we detect sharp elevation increase/decrease of up to 25 meters at time-steps 23-24 which corresponds to the transition between ERS-1/2 data. As a result, ERS-1 kriged results were temporarily removed from our
analyses of $dh/dt$ reported above for regions C1500908272 and C0100408171. It is unclear what causes the discrepancy in ERS-1/2 in the blocks in these two regions. Uncertainties in the slope $dh/dt$ estimates were obtained from the RMS deviations from trend and the assumption that the RMS (noise) is statistically independent in time. This assumption is violated if our estimations of the annual/semi-annual or any time-dependent (natural or systematic) signals are incorrect, in which case the uncertainty estimates serve only as a lower bound.

**Work in progress**

The RMS and error bars in $dh/dt$ in this study are affected mostly by the poor quality of GTOPO30 DEMs in the polar regions. Our original intention was to krig on topography residuals so that changes are with respect to near-zero baselines. However it appears that inappropriate DEMs are the limiting factors. We are now in the process of testing the method on both the original ERS data, and residuals between ERS data and the 5-km DEM produced by Bamber et al. [1996]. With high quality ERS data over the Amery Shelf as compared with GPS data in Phillips [1997], we anticipate an improvement in error estimates of $dh/dt$. More importantly we wish to separate as much as possible uncertainties in the topography field itself from those due to additional uncertainties in the data/DEMs.

In addition we will investigate the significance of anisotropies in $\square$ and $\Box$, i.e., whether using anisotropic parameters would make statistically significant different results compared to those of the isotropic cases. The relation between the anomalous lows in $\square$-profiles and track directions will be investigated. Tidal corrections over the Amery Shelf will be explored further. Our long term goal is to develop a method to study changes both in the interior and at the edges of the ice sheets. With ICESat data, we hope to be able to obtain high quality DEMs and improve spatial resolution at small distances in the modeled variograms. In addition we expect to be able to assess the validity of the linear trends observed in our ERS results.

**Summary**

We analyzed topography residuals from ERS-1/2 radar altimetry data with GTOPO30 DEM to produce a map of elevation changes and RMS deviations from linear trends for Antarctica. After making various corrections as described in Table.2, data mean and variance remain stable with time within each studied region. Variogram analyses show anisotropic $\square$ and $\Box$ parameters that could be related to topographic gradients in West Antarctica and to along-track directions in East Antarctica. Time sequences of kriged blocks at approximate resolution $100km \times 100km$ shows a dominant annual signals of amplitude ranges [$0.13m,0.36m$] which could be partially explained by firn compaction [Zwally et al., 2002]. After removals of the annual signals, fitted linear trends to time sequence indicate near-zero height changes in the interior eastern part of Antarctica, positive $dh/dt$ of $47\pm40 \text{ cm/yr}$ to $96\pm62 \text{ cm/yr}$ at the head of the Amery Shelf. $Dh/dt$ and RMS deviations from trends are small in the interior part of East Antarctica, and large along the coast as expected. Positive changes in West Antarctica of $30\pm4 \text{ cm/yr}$ to $14\pm4 \text{ cm/yr}$ near [-81,-120] are consistent with those obtained from cross-over analyses [Zwally et al., 2002]. The limiting factor in determining uncertainties in $dh/dt$ in this method comes from the GTOPO30 DEMs. We are currently working on improving the uncertainties using both improved DEMs to form topography residuals and working only with original ERS data.
References

<icesat4.gsfc.nasa.gov>}
Nguyen, A.T. and T.A. Herring, Analysis of ERS-1 Radar Altimetry data using Kriging, Fall 2002 AGU.
### Table 1 - ERS Data

<table>
<thead>
<tr>
<th>[start, end]</th>
<th>time-step</th>
<th>Phase</th>
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<tr>
<td>[940417, 941008]</td>
<td>1-5</td>
<td>E (168-day repeat)</td>
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<tr>
<td>[941009, 950401]</td>
<td>6-11</td>
<td>F (168-day repeat)</td>
</tr>
<tr>
<td>[950402, 960602]</td>
<td>12-23</td>
<td>G (35-day repeat)</td>
</tr>
<tr>
<td>[960601, 011231]</td>
<td>24-49</td>
<td>35-day repeat</td>
</tr>
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### Table 2 - Possible Errors in Radar Altimetry over the Ice Sheets

<table>
<thead>
<tr>
<th>Source</th>
<th>Range (m)</th>
<th>Correction (m)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tropospheric delays</td>
<td>1.5 to 2.5</td>
<td>GSFC - ECMWF (^2)</td>
<td><a href="http://icesat4.gsfc.nasa.gov/">http://icesat4.gsfc.nasa.gov/</a></td>
</tr>
<tr>
<td>Ionospheric delays</td>
<td>0.02 to 0.10</td>
<td>GSFC - IR90 (^3)</td>
<td><a href="http://icesat4.gsfc.nasa.gov/">http://icesat4.gsfc.nasa.gov/</a></td>
</tr>
<tr>
<td>Orbit</td>
<td>-0.3 to 0.25</td>
<td>GSFC - DGM-E04 (^5)</td>
<td>ERS-1 data, Fricker et al. [2002]</td>
</tr>
</tbody>
</table>

\(^1\) Goddard Space Flight Center, Ice Altimetry group  
\(^2\) European Center for Medium Range Weather Forecasts  
\(^3\) International Reference Ionosphere 1999  
\(^4\) University of Texas Center for Space Research tide model developed by Richard Eanes  
\(^5\) Delft University DGM-E04 gravity model
Figure 1: Antarctica contour map
Elevation contour map of Antarctica (in km) showing gridded blocks used in this study.
Figure 2: Variogram models - three scenarios
Variogram models cases A (low $\sigma^2$, high $\gamma(h)$), C (high $\sigma^2$, low $\gamma(h)$), and B (transition between A and C). $\sigma^2$ is the variance of topography residuals, $\gamma(h)$ the variogram as a function of the distance $h$. The dashed line represents uniformly random distributed residuals with no correlation length. Case A implies a strong correlation between topography residuals at very small $h$ (low $\gamma(h)$) and that correlation length is high (high $\sigma^2$). Case C implies very little correlation at small $h$ (high $\sigma^2$) and very short correlation length (low $\gamma(h)$). This also implies potential nugget effects.
Figure 3: Variogram as a function of azimuth
Azimuthal dependence of inverted variogram parameters $\alpha$ & $\beta$ for the four regions $C1500908272$ (a-b), $C0100408171$ (c-d), $C0500908171$ (e-f), $C1001408167$ (g-h). The isotropic case is shown at azimuth N-15E. $\alpha$ appears strongly anisotropic for two regions $C1500908272$ and $C0500908171$, and $\beta$ has weaker anisotropy (relative to $\alpha$) and is dominant in regions $C1500908272$ and $C0100408171$. 
Figure 4: Kriged results - Slope

Linear trends implying rate of change $dh/dt$ (cm/yr) of elevation in Antarctica. Small changes in the interior part of eastern Antarctica indicate the region is stable. In contrast, large changes, both positive and negative in western Antarctica suggest instability. More consistent positive changes of $47\pm40$ cm/yr to $96\pm62$ cm/yr are found at the head of the Amery Ice Shelf, and also near the coast at $0^\circ$ longitude.
Figure 5: Kriged results - RMS
RMS deviations from linear trends for kriged blocks. Small RMS is found in the stable interior part of eastern Antarctica. The large RMS near the coastal region, head of the Amery Ice Shelf, and in west Antarctica indicate a combination of instability, large measurement errors and insufficient a-priori DEMs.